CSL 105: Discrete Mathematical Structures
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1. Given a list of n natural numbers $d_1, ..., d_n$, show how to decide in polynomial time whether there exists an undirected graph $G = (V, E)$ whose vertex degrees are precisely $d_1, ..., d_n$. (That is, if $V = \{v_1, ..., v_n\}$, then the degree of v_i should be exactly d_i.) G should not contain mtultiple edges between the same pair of nodes, or "loop" edges (where both end vertices are the same node).

2. Suppose that 21 girls and 21 boys enter a mathematics competition. Furthermore, suppose that each entrant solves at most six questions, and for every boy-girl pair, there is at least one question that they both solved. Show that there is a question that was solved by at least three girls and at least three boys.

3. How many ways are there for a horse race with four horses to finish if ties are possible?

4. Prove the *Hockeystick identity*
$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}$$
whenever n and r are positive integers,
   (a) using a combinatorial argument.
   (b) using Pascal's identity.

5. Give a combinatorial proof that $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$.

6. Give a combinatorial proof that $\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$.

7. Suppose that a weapons inspector must inspect each of five different sites twice, visiting one site per day. The inspector is free to select the order in which to visit these sites, but cannot visit site X, the most suspicious site, on two consecutive days. In how many different orders can the inspector visit these sites?

8. Consider the following two theorems:

   Theorem 1 (Permutation with indistinguishable objects) *The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, ..., and n_k indistinguishable objects of type k, is*
$$\frac{n!}{n_1! n_2! ... n_k!}.$$

Theorem 2 (Distinguishable objects into distinguishable boxes) *The number of ways to distribute n distinguishable objects into k distinguishable boxes so that n_i objects are placed into box i, $i = 1, 2, ..., k$, equals*

$$\frac{n!}{n_1!n_2!...n_k!}$$

Prove the second theorem by first setting up a one-to-one correspondence between permutations of n objects with n_i indistinguishable objects of type $i, i = 1, 2, ..., k$ and the distribution of n objects in k boxes such that n_i objects are placed in box $i, i = 1, 2, ..., k$ and then applying the first theorem.