CSL 105: Discrete Mathematical Structures
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1. Discuss homework-2.

2. A jigsaw puzzle is put together by successively joining pieces that fit together into blocks. A move is made each time a piece is added to a block, or when two blocks are joined. Use strong induction to prove that no matter how the moves are carried out, exactly $n-1$ moves are required to assemble a puzzle with n pieces.

3. Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.
   Basis step: $a^0 = 1$ is true by the definition of a^0.

   Inductive step: Assume that $a^j = 1$ for all nonnegative integers j with $j \leq k$. Then note that $a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1$.

4. Given a Bipartite graph $G = (X, Y, E)$, a matching in G is defined to be a subset of $M \subseteq E$ such that for any $(x, y) \in M$, both x and y do not appear as the end vertex of any edge in M. Given $|X| = |Y| = n$, a perfect matching of G is a matching of size n. Show the following theorem:

   Theorem 1 (Hall's Theorem) *Given a bipartite graph $G = (X, Y, E)$ such that $n = |X| = |Y|$, there is a perfect matching in G is and only if for ever subset of vertices $S \subseteq X, |N(S)| \geq |S|$, where $N(S)$ denotes the set of neighboring vertices of S.*

5. Given a list of n natural numbers $d_1, ..., d_n$, show how to decide in polynomial time whether there exists an undirected graph $G = (V, E)$ whose vertex degrees are precisely $d_1, ..., d_n$. (That is, if $V = \{v_1, ..., v_n\}$, then the degree of v_i should be exactly d_i.) G should not contain mtultiple edges between the same pair of nodes, or "loop" edges (where both end vertices are the same node).