
CSL 105: Discrete Mathematical Structures**Instructor:** Ragesh Jaiswal

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1. (**Universal Hashing**) Hashing is a technique used to store elements from a large universe $U = \{0, \dots, m - 1\}$ using a small table $T = \{0, \dots, n - 1\}$ using a hash function $h : U \rightarrow T$ such that the number of collisions are minimized ¹.

Using a fixed hash function might not work. So, we use a *family* of hash functions H and then pick a hash function randomly from this family. A hash function family H is called 2-universal if

$$\forall x, y \in U, x \neq y, \Pr_{h \leftarrow H}[h(x) = h(y)] \leq 1/n.$$

Show how a 2-universal hash function family is useful in hashing and give an example of such a family.

2. Show that if L and M are regular languages, then so is $L \cap M$.
3. Show that the following languages are not regular:
 - $L = \{0^n \mid n \text{ is a perfect square}\}$
 - $M = \{w \mid w \in \{0, 1\}^* \text{ and } w \text{ is a palindrome}\}$

¹Assume that collisions are resolved using auxiliary data structure