CSL 105: Discrete Mathematical Structures
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1. Remaining problems from previous tutorial.

2. k objects are picked independently at random with replacement from a set of n distinct objects. For $1 \leq i < j \leq k$, let X_{ij} denote the indicator random variable that is 1 if the i^{th} and j^{th} objects are the same otherwise 0. Show that for any $i < j$ and $p < q$ such that $(i, j) \neq (p, q)$, the random variables X_{ij} and X_{pq} are independent.

3. (**Coupon-collector problem**) Every time you go to the superstore, you get a random coupon out of n distinct coupons. How many times do you have to visit the store to be able to collect all distinct coupons?

4. (**Balls and bins**) n balls are thrown randomly into n bins. What is the maximum number of balls that any bin can have?

5. (**Universal Hashing**) Hashing is a technique used to store elements from a large universe $U = \{0, ..., m-1\}$ using a small table $T = \{0, ..., n-1\}$ using a hash function $h : U \rightarrow T$ such that the number of collisions are minimized [1].

   Using a fixed hash function might does not work. So, we use a *family* of hash functions H and then pick a hash function randomly from this family. A hash function family H is called 2-universal if

   $$\forall x, y \in U, x \neq y, \mathbf{Pr}_{h \leftarrow H}[h(x) = h(y)] \leq 1/n.$$

   Show how a 2-universal hash function family is useful in hashing and give an example of such a family.

[1] Assume that collisions are resolved using auxiliary data structure