CSL 105: Discrete Mathematical Structures
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1. Consider the following two theorems:

   Theorem 1 (Permutation with indistinguishable objects) *The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type $2, ...,$ and n_k indistinguishable objects of type k, is*

   $$\frac{n!}{n_1!n_2!...n_k!}.$$

   Theorem 2 (Distinguishable objects into distinguishable boxes) *The number of ways to distribute n distinguishable objects into k distinguishable boxes so that n_i objects are placed into box i, $i = 1, 2, ..., k$, equals*

   $$\frac{n!}{n_1!n_2!...n_k!}$$

   Prove the second theorem by first setting up a one-to-one correspondence between permutations of n objects with n_i indistinguishable objects of type $i, i = 1, 2, ..., k$ and the distribution of n objects in k boxes such that n_i objects are placed in box $i, i = 1, 2, ..., k$ and then applying the first theorem.

2. In how many ways can you place n distinguishable objects into k indistinguishable boxes?

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4. If k is an integer with $k \geq 2$, then $R(k, k) \geq 2^{k/2}$.

5. Let $G = (V, E)$ be a graph on n vertices and $m \geq n/2$ edges. Then G has an independent set of size at least $n^2/4m$.

6. James Bond is imprisoned in a cell from which there are three possible ways to escape: an air- conditioning duct, a sewer pipe and the door (which is unlocked). The air-conditioning duct leads him on a two-hour trip whereupon he falls through a trap door onto his head, much to the amusement of his captors. The sewer pipe is similar but takes five hours to traverse. Each fall produces temporary amnesia and he is returned to the cell immediately after each fall. Assume that he always immediately chooses one of the three exits from the cell with probability 1/3 . On the average, how long does it take before he realizes that the door is unlocked and escapes?