> • **Use of unfair means will be severely penalized.**

There are 3 questions for a total of 50 points.

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(10) 1. Let D_n denote the number of derangements of n objects. Show that $D_n = n \cdot D_{n-1} + (-1)^n$ for $n \geq 1$.

2. Give DFA's accepting the following languages over the alphabet $\{0, 1\}$:

(10)   (a) The set of all strings beginning with a 1 that, when interpreted as a binary integer, is a multiple of 5. For example, strings 101, 1010, and 1111 are in the language and 0, 100, and 111 are not.

(10)   (b) The set of all strings that, when interpreted in reverse as a binary integer, is divisible by 5. Example of strings in the language are 0, 10011, 1001100, and 0101.

3. Prove that the following languages are not regular:

(10)   (a) The set of strings of 0's and 1's, beginning with 1, such that when interpreted as a binary integer, the integer is a prime.

(10)   (b) The set of strings of the form $0^i 1^j$ such that the GCD of i and j is 1.