## • Use of unfair means will be severely penalized.

There are 4 questions for a total of 50 points.

- (10) 1. A line of n airline passengers is waiting to board a plane. They each hold a ticket to one of the 100 seats on that flight. The  $i^{th}$  passenger in line has a ticket for the seat number i. Unfortunately, the first person in line is crazy, and will ignore the seat number on his ticket, picking a random seat to occupy. All the other passengers are quite normal, and will go to their proper seat unless it is already occupied. If their given seat is occupied, they will then find a free seat to sit in, (uniformly) at random. What is the probability that the last  $(n^{th})$  person to board the plane will sit in his/her proper seat (i.e., seat number n)?
- (15) 2. Show that the average-case complexity of the quick-sort algorithm is  $O(n \log n)$ .
- (15) 3. For certain application, we need to generate a set X of positive integers such that  $\forall p, q \in X, p + q \notin X$ . Let us call such sets *nice*. Show that for any set X of positive integers, there is a subset  $Y \subseteq X$  such that Y is nice and  $|Y| \ge |X|/3$ .
- (10) 4. We select two points uniformly independently from a line segment of unit length. The two points partition the segment into three sub-segments.
  - (a) What is the expected length of the smallest segment?
  - (b) What is the probability that the three sub-segments can form a triangle?