

- Use of unfair means will be severely penalized.

There are 4 questions for a total of 50 points.

- (10) 1. A graph is said to be connected if there is a path between any pair of vertices. Show that for any graph G , either G or its complement \bar{G} (includes only those edges that are not in G) is connected.
2. Solve the following two questions related to the “big-oh” notation.
- (5) (a) Determine whether $\log(n!)$ is $\theta(n \log n)$. Justify your answer.
- (5) (b) Show that n^n is not $O(n!)$.
- (15) 3. Recall the Euclid-GCD(a, b) algorithm discussed in the lectures for finding the gcd of two integers a and b . Prove the following theorem:
- Theorem 1** (Lame’s theorem). *For any integer $k \geq 1$, if $a > b \geq 1$ and $b < F_{k+1}$, then the call Euclid-GCD(a, b) makes fewer than k recursive calls.*
- Here F_k denotes the k^{th} number in the Fibonacci sequence $(0, 1, 1, 2, 3, 5, 8, 13, \dots)$
- (15) 4. Design an algorithm that takes as input positive integers a, b, m and outputs $a^b \pmod{m}$ (input/output is in binary). Discuss the worst-case time complexity of your algorithm. Note that for this problem, the better the running time, the more points you will get.