- **Use of unfair means will be severely penalized.**

There are 5 questions for a total of 50 points.

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(6) 1. Use resolution principle to show that the following argument is valid:

1. If we buy a CD player, we must cut down on small purchases or our financial plans will be ruined.
2. If we buy more compact discs, we will not be cutting down on small purchases.
3. But if we buy a CD player, we will buy more compact discs.

Therefore, if we buy a CD player, our financial plans will be ruined.

(8) 2. Prove or disprove the following statements:

1. $\forall x\ (P(x) \wedge Q(x))$ is equivalent to $(\forall x\ P(x)) \wedge (\forall x\ Q(x))$.
2. $\exists x\ (P(x) \wedge Q(x))$ is equivalent to $(\exists x\ P(x)) \wedge (\exists x\ Q(x))$.
3. $\forall x\ (P(x) \vee Q(x))$ is equivalent to $(\forall x\ P(x)) \vee (\forall x\ Q(x))$.
4. $\forall x\ (P(x) \vee Q(x))$ is equivalent to $(\exists x\ P(x)) \vee (\exists x\ Q(x))$.

(6) 3. Consider the statement,

> The product of any irrational number and any rational number is irrational.

Is the following proposed negation of this statement the correct negation? If not, what is the correct negation?

> There exists an irrational number x and an irrational number y such that the product xy is rational.

Which is true, the original statement or its negation? Give appropriate proof.

(10) 4. Prove or disprove: If a, b, c are odd integers, then the equation $ax^2 + bx + c = 0$ has no rational solution.

(20) 5. A bipartite graph $G = (V, E)$ is a graph where the vertices can be partitioned into two non-empty subsets V_1 and V_2 such that there does not exist an edge between any two vertices of V_1 and the same holds for V_2. A cycle in a graph is a sequence of vertices $v_1, ..., v_k$ such that $v_1 = v_k$ and vertices $v_1, ..., v_{k-1}$ are distinct. Furthermore, for all $1 \leq i < k$, there is an edge between vertex v_i and v_{i+1}.

Prove or disprove the following:

- For any given graph, the graph is a bipartite graph iff there is no odd length cycle in the graph.