There are 5 questions for a total of 50 points.

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(10) 1. Prove that $\mathbf{NP} \subseteq \mathbf{BPP}$ implies that $\mathbf{RP} = \mathbf{NP}$.

(10) 2. Prove that Chernoff bounds hold for arbitrary random variables in the $[0, 1]$ interval. Note that we showed the bounds for 0/1 variables in the class.

(10) 3. Consider the "balls and bins" problem where n balls are to be thrown into n bins. Suppose we consider the d-choice process instead of 2-choice. That is, for each ball we randomly select d bins and assign the ball into the bin with least load. Show that with high probability the most loaded bin has a load of $O\left(\frac{\log \log n}{\log d}\right)$.

(10) 4. Consider the following version of the "balls and bins" problem: There are n bins to start with and balls are thrown in multiple rounds. In the first round n balls are thrown into n bins randomly. After the first round, the bins having at least one ball are removed. In the second round n balls are thrown randomly into the remaining bins. Again, after the second round, the non-empty bins are removed and in the thirds round n balls are thrown into the remaining bins. Show that the expected number of rounds for which this process runs is $c \cdot \log^* n$ (for some constant c).

(10) 5. Let $G = (V, E)$ be an undirected graph and suppose each $v \in V$ is associated with a set $S(v)$ of $8r$ colours, where $r \geq 1$. Suppose, in addition, that for each $v \in V$ and $c \in S(v)$ there are at most r neighbors u of v such that c lies in $S(u)$. Prove that there is a proper colouring of G assigning to each vertex v a colour from its class $S(v)$ such that, for any edge $(u, v) \in E$, the colors assigned to u and v are different.