CSL 356: Analysis and Design of Algorithms **Date:** August 23, 2012
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1. Let T be an minimum spanning tree of graph G. Given a connected subgraph H of G, show that $T \cap H$ is contained in some minimum spanning tree of H.

2. Prove the following two properties of the Huffman encoding scheme:

   (a) If some character occurs with frequency more than $2/5$, then there is guaranteed to be a codeword of length 1.

   (b) If all characters occur with frequency less than $1/3$, then there is guaranteed to be no codeword of length 1.

3. There are n jobs that are supposed to be scheduled on a single machine. With each job i, there is an associated duration $t(i)$ that denotes the time that job i will take to execute on the machine. Each job i also has a given weight $w(i)$ that denotes the importance of this job. Given a schedule (an ordering of jobs to be executed on the machine), the completion time of job i is the time at which this job is finished by the machine. Let the completion time of job i be denoted by $C(i)$. Design an algorithm for giving a schedule that minimizes $\sum_{i=1}^{n} w(i) \cdot C(i)$. Give proof of correctness and discuss running time.

4. Argue that the approximation factor of $O(\log n)$ given by the greedySetCover algorithm, discussed in class for the Set cover problem, is tight. What this means is that you have to think of problem instances where the greedy algorithm gives an approximation factor of $\Omega(\log n)$.