CSL 356: Analysis and Design of Algorithms **Date:** August 6, 2012
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1. Consider two vertices s and t in a given undirected graph. A vertex u (different from s and t) is called critical with respect to s and t if the removal of u from the graph disconnects s and t. Suppose in a given graph the shortest distance between s and t is strictly greater than $\lceil n/2 \rceil$. Prove or disprove the following statement:

   There exists a vertex that is critical with respect to s and t.

   Give an algorithm for finding this vertex in case there exists one.

2. You are a party organizer and you need to solve the following problem. There are n people and you know their friendship network. Your job is to decide a subset S of people who will be invited to the party. The constraint that you need to satisfy is that every person in the subset S, is friends with at least five other people in S **and** not friends with at least five other people in S. Assume that you are given the friendship network as a graph (assume adjacency list representation) where the edges denote friendships. Design an algorithm that maximizes the size of the set S.

3. This is problem number 29, chapter 4 from the Tardos Kleinberg book.

   Given a list of n natural numbers $d_1, ..., d_n$, show how to decide in polynomial time whether there exists an undirected graph $G = (V, E)$ whose vertex degrees are precisely $d_1, ..., d_n$. (That is, if $V = \{v_1, ..., v_n\}$, then the degree of v_i should be exactly d_i.) G should not contain mtultiple edges between the same pair of nodes, or "loop" edges (where both end vertices are the same node).