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## CSL 356: Analysis and Design of Algorithms

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### 1. Another algorithm for max-flow

Consider the following slightly changed version of the Ford-Fulkerson max-flow algorithm. This algorithm is also due to Jack Edmonds and Richard Karp.

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Max-Flow

- Start with a flow  $f$  such that  $\forall e \in E, f(e) = 0$ .
- While there is an  $s - t$  path in  $G_f$ 
  - Find an  $s - t$  path  $P$  in  $G_f$  with **largest bottleneck value**.
  - Augment along  $P$  to obtain  $f'$ .
  - Update  $f$  to  $f'$  and  $G_f$  to  $G_{f'}$

return( $f$ ).

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- (a) Think of an algorithm to find the *largest bottleneck path* from  $s$  to  $t$  in a given graph. A bottleneck path is a path such that the bottleneck edge has maximum weight. Discuss its running time.  
(*Hint: Try ideas from Dijkstra's Algorithm.*)
- (b) Let  $f$  be any  $s - t$  flow and  $t$  be the value of maximum flow in the residual graph  $G_f$ . Let  $f'$  be the new flow after one augmentation and  $t'$  be the value of the new maximum flow in the residual graph  $G_{f'}$ . Argue that  $t' \leq (1 - 1/m) \cdot t$ .
- (c) Use the properties you showed above to argue that for a graph with integer capacities, the algorithm runs in time  $O(m^2 \cdot \log m \cdot \log f^*)$ , where  $f^*$  is the value of the max-flow in the original graph  $G$ .

2. Discuss problems in Homework-3.