

- Always try to give algorithm with best possible running time. The points that you obtain will depend on the running time of your algorithm. For example, a student who gives an $O(n)$ algorithm will receive more points than a student who gives an $O(n^2)$ algorithm.
- You are required to give proofs of correctness whenever needed. For example, if you give a greedy algorithm for some problem, then you should also give a proof why this algorithm outputs optimal solution.

- **Use of unfair means will be severely penalized.**

There are 3 questions for a total of 50 points.

- (20) 1. Given an undirected graph $G = (V, E)$, we call a vertex critical if its removal disconnects the graph. Give an algorithm that finds all the critical vertices in the graph and that has a running time of $O(|V| + |E|)$.
- (15) 2. Given a weighted, undirected graph G and a minimum spanning tree T of G . Suppose that we decrease the weight of one of the edges not in T . Give an algorithm for finding the minimum spanning tree in the modified graph.
- (15) 3. There is a currency system that has coins of value $1, c, c^2, c^3, \dots, c^k$ for some integers $c, k > 1$. You have to pay a person V units of money using this currency. Your goal is to minimize the total number of coins. Design an algorithm that minimizes the total number of coins.