Course Information

CSE 207 — Modern Cryptography

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Cryptography usage

Did you use any cryptography

- today?

Cryptography usage

Did you use any cryptography

- today?
- over the last week?

Cryptography usage

Did you use any cryptography

- today?
- over the last week?
- over the Christmas break?

Cryptography usage

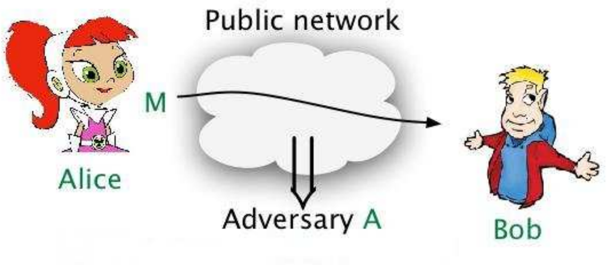
- https invokes the Secure Socket Layer (SSL) communication security protocol to securely transmit your credit card number to the server
- SSL uses cryptography

Cryptography usage

Other uses of cryptography

- ATM machines
- On-line banking
- Remote login and file transfer using SSH

What is cryptography about?

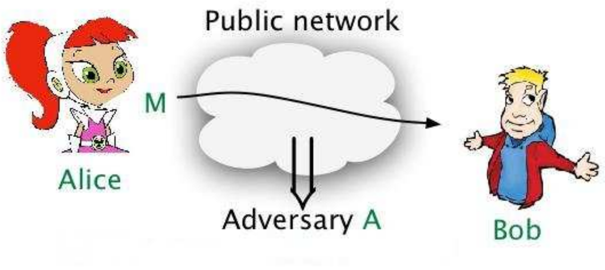


Adversary: clever person with powerful computer

Goals:

- Data privacy
- Data integrity and authenticity

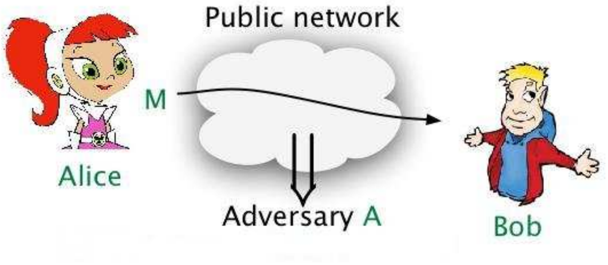
Privacy



The goal is to ensure that the adversary does not see or obtain the data (message) M.

Example: M could be a credit card number being sent by shopper Alice to server Bob and we want to ensure attackers don't learn it.

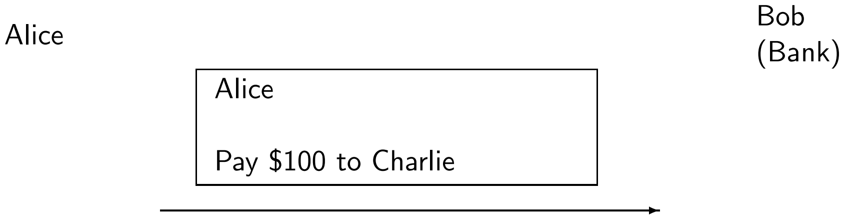
Integrity and authenticity



The goal is to ensure that

- M really originates with Alice and not someone else
- M has not been modified in transit

Integrity and authenticity example



Adversary Eve might

- Modify “Charlie” to “Eve”
- Modify “$100” to “$1000”

Integrity prevents such attacks.

Medical databases

Doctor | **Database**

Get Alice →

Alice	F_A
Bob	F_B

← F_A

Reads F_A

Modifies F_A to F'_A

Put: Alice, F'_A →

Alice	F'_A
Bob	F_B

Medical databases

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Put: Alice, F'_A →

Alice	F'_A
Bob	F_B

- Privacy: F_A, F'_A contain confidential information and we want to ensure the adversary does not obtain them
- Integrity and authenticity: Need to ensure
  - doctor is authorized to get Alice's file
  - F_A, F'_A are not modified in transit
  - F_A is really sent by database
  - F'_A is really sent by (authorized) doctor

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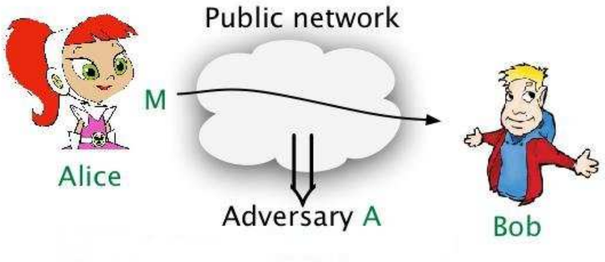

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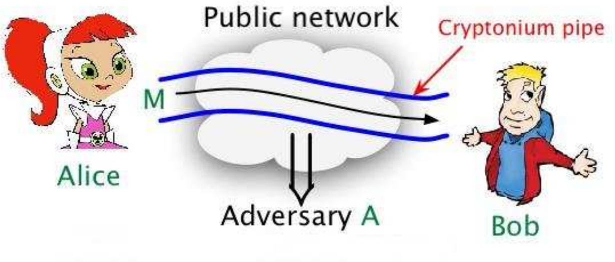
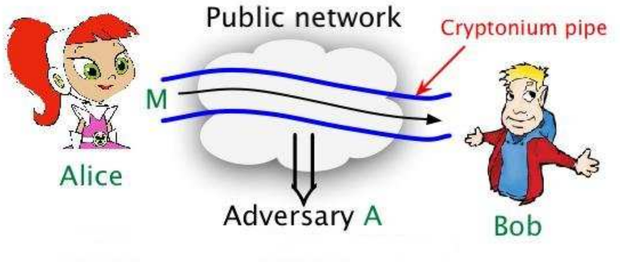
Ideal World



Cryptonium pipe: Cannot see inside or alter content.

Ideal World



Cryptonium pipe: Cannot see inside or alter content.

All our goals would be achieved!

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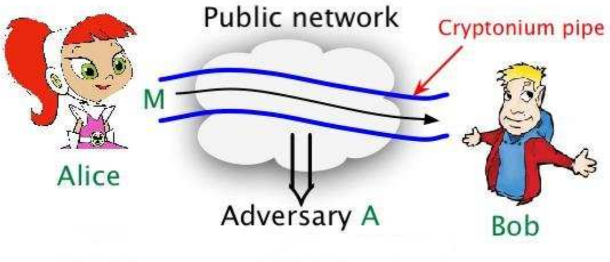
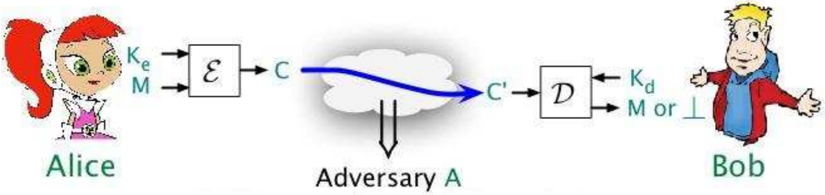


Cryptonium pipe: Cannot see inside or alter content.

All our goals would be achieved!

But cryptonium is only available on planet Crypton and is in short supply. ☹

Cryptographic schemes



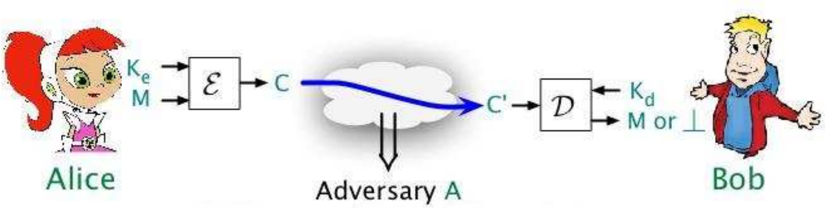
$\mathcal{E}$: encryption algorithm

$\mathcal{D}$: decryption algorithm

K_e: encryption key

K_d: decryption key

Cryptographic schemes

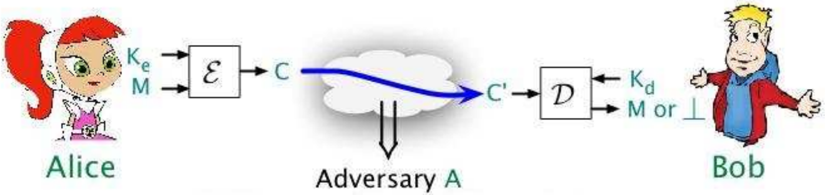


$\mathcal{E}$: encryption algorithm K_e: encryption key

$\mathcal{D}$: decryption algorithm K_d: decryption key

Algorithms: standardized, implemented, public!

Cryptographic schemes



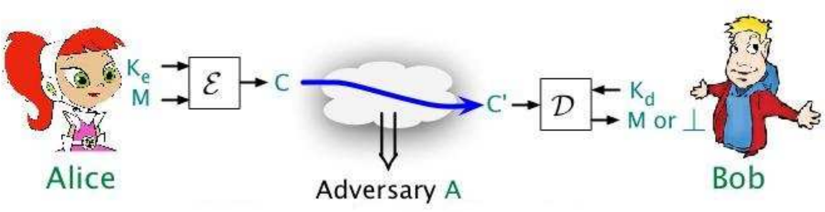
$\mathcal{E}$: encryption algorithm K_e: encryption key

$\mathcal{D}$: decryption algorithm K_d: decryption key

Settings:

- public-key (assymmetric): K_e public, K_d secret
- private-key (symmetric): $K_e = K_d$ secret

Cryptographic schemes



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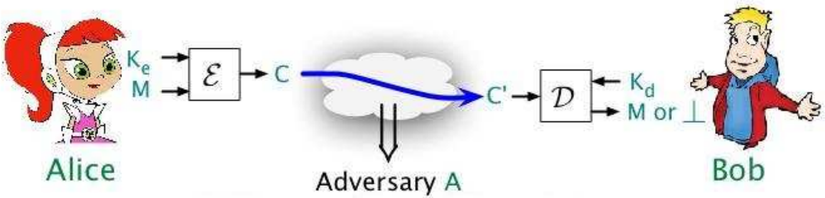
How do keys get distributed? Magic, for now!

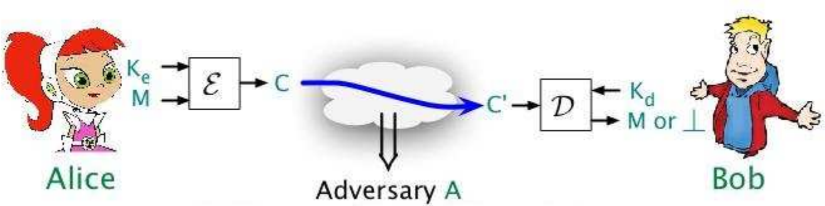
Cryptographic schemes



Our concerns:

- How to define security goals?
- How to design $\mathcal{E}$, $\mathcal{D}$?
- How to gain confidence that $\mathcal{E}$, $\mathcal{D}$ achieve our goals?

Cryptographic schemes



Computer Security: How does the computer/system protect K_e/K_d from break-in (viruses, worms, OS holes, ...)? (CSE 127,227)

Cryptography: How do we use K_e, K_d to ensure security of communication over an insecure network? (CSE 107,207)

Why is cryptography hard?

- One cannot anticipate an adversary strategy in advance; number of possibilities is infinite.
- "Testing" is not possible in this setting.

Early history

Substitution ciphers/Caesar ciphers:

$$K_e = K_d = \pi : \Sigma \to \Sigma, \text{a secret permutation}$$

e.g., $\Sigma = \{A, B, C, \ldots\}$ and π is as follows:

σ	A	B	C	D	$\cdots$
$\pi(\sigma)$	E	A	Z	U	$\cdots$

$$\begin{aligned} \mathcal{E}_\pi(CAB) &= \pi(C)\pi(A)\pi(B) \\ &= Z\ E\ A \\ \mathcal{D}_\pi(ZEA) &= \pi^{-1}(Z)\pi^{-1}(E)\pi^{-1}(A) \\ &= C\ A\ B \end{aligned}$$

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Not very secure! (Common newspaper puzzle)

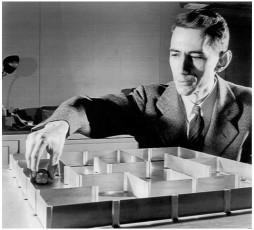
The age of machines

Enigma: German World War II machine

Broken by British in an effort led by Turing

Shannon and One-Time-Pad (OTP) Encryption

$$K_e = K_d = \underbrace{K \stackrel{\$}{\leftarrow} \{0,1\}^k}_{K \text{ chosen at random from } \{0,1\}^k}$$



For any $M \in \{0,1\}^k$

- $\mathcal{E}_K(M) = K \oplus M$
- $\mathcal{D}_K(C) = K \oplus C$

Shannon and One-Time-Pad (OTP) Encryption

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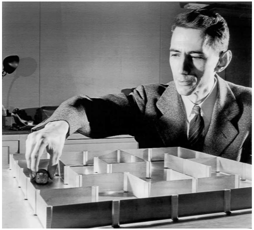

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Theorem (Shannon): OTP is perfectly secure as long as only one message encrypted.

"Perfect" secrecy, a notion Shannon defines, captures mathematical impossibility of breaking an encryption scheme.

Fact: if $|M| > |K|$, then no scheme is perfectly secure.

Modern Cryptography: A Computational Science

Security of a "practical" system must rely not on the impossibility but on the computational difficulty of breaking the system.

("Practical" = more message bits than key bits)

Modern Cryptography: A Computational Science

Rather than:

"It is impossible to break the scheme"

We might be able to say:

"No attack using $\leq 2^{160}$ time succeeds with probability $\geq 2^{-20}$"

I.e., Attacks can exist as long as cost to mount them is prohibitive, where Cost = computing time/memory, $$$

Modern Cryptography: A Computational Science

Security of a "practical" system must rely not on the impossibility but on the computational difficulty of breaking the system.

Cryptography is now not just mathematics; it needs to draw on computer science

- Computational complexity theory (CSE 105,200)
- Algorithm design (CSE 101,202)

Classical Approach: Iterated design

Scheme 1.1

Classical Approach: Iterated design

Scheme 1.1 $\longrightarrow$ bug!

Classical Approach: Iterated design

Scheme 1.1 $\longrightarrow$ bug!

$\downarrow$

Scheme 1.2

Classical Approach: Iterated design

Scheme 1.1	$\longrightarrow$	bug!
$\downarrow$		
Scheme 1.2	$\longrightarrow$	bug!

Classical Approach: Iterated design

$$\begin{array}{ccc} \text{Scheme 1.1} & \longrightarrow & \text{bug!} \\ \downarrow & & \\ \text{Scheme 1.2} & \longrightarrow & \text{bug!} \\ \downarrow & & \\ \vdots & & \\ \downarrow & & \\ \text{Scheme 1.}n & & \end{array}$$

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Scheme 1.n $\longrightarrow$ deploy

Classical Approach: Iterated design

Scheme 1.1 $\rightarrow$ bug!

$\downarrow$

Scheme 1.2 $\rightarrow$ bug!

$\downarrow$

$\vdots$

$\downarrow$

Scheme 1.n $\rightarrow$ deploy $\rightarrow$ bug!

Good cryptography

- Understanding the goals: Formal adversarial models and definitions of security goals

- Beyond iterated design: Proof by reduction that a construction achieves its goal

Defining security

A great deal of design tries to produces schemes without first asking:

"What exactly is the security goal?"

This leads to schemes that are complex, unclear, and wrong.

Defining security

Being able to precisely state what is the security goal of a design is challenging but important.

We will spend a lot of time developing and justifying strong, precise notions of security.

Thinking in terms of these precise goals and understanding the need for them may be the most important thing you get from this course!

The factoring problem

Input: Composite integer N
Desired output: prime factors of N

Example:

Input: 85

Output:

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Can we write a factoring program? Easy!

Alg Factor(N) // N a product of 2 primes
For $i = 2, 3, \ldots, \lceil \sqrt{N} \rceil$ do
If $N \bmod i = 0$ then return i

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But this is very slow ... 
Prohibitive if N is large (e.g., 400 digits)

Can we factor fast?

- Gauss couldn't figure out how
- Nor does anyone know now

Nobody today knows how to factor a 400 digit number in a practical amount of time.

Provable Security

Provide

- A scheme
- A proof of security

The proof establishes something like:

> "The only way to break the scheme is to factor a large number"

or, put another way

> "If an adversary breaks the scheme, it must have found a fast factoring algorithm."

Provable Security

Bug in scheme implies

- attacker has found a way to factor fast
- attacker is smarter than Gauss
- and smarter than all living mathematicians...

Atomic Primitives or Problems

Examples:

- Factoring: Given large $N = pq$, find p, q
- Block cipher primitives: DES, AES, ...
- Hash functions: MD5, SHA1, ...

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Features:

- Few such primitives
- Bugs rare
- Design an art, confidence by history.

Atomic Primitives or Problems

Examples:

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- Block cipher primitives: DES, AES, ...
- Hash functions: MD5, SHA1, ...

Features:

- Few such primitives
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Drawback: Don't directly solve any security problem.

Higher Level Primitives

Goal: Solve security problem of direct interest.

Examples: encryption, authentication, digital signatures, key distribution, . . .

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Examples: encryption, authentication, digital signatures, key distribution, . . .

Features:

- Lots of them
- Bugs common in practice

Lego Approach

We typically design high-level primitives from atomic ones

Atomic primitive

↓

Transformer

↓

High-level primitive

History shows that the Transformer is usually the weak link:

- Atomic primitives secure, yet
- Higher level primitive insecure

Provable security

Enables us to get transformers for which we can guarantee

Atomic primitive secure $\Rightarrow$ High-level primitive secure

I.e., If attacker breaks encryption scheme then they are smarter than Gauss.

Provable security in practice

Proven-secure schemes in use (SSL, SSH, IPSec, . . .):

- HMAC
- OAEP
- ECIES
- . . .

New uses for old mathematics

Cryptography uses

- Number theory
- Combinatorics
- Modern algebra
- Probability theory

Modern Cryptography: Esoteric mathematics?

Hardy, in his essay A Mathematician's Apology writes:

> *"Both Gauss and lesser mathematicians may be justified in rejoicing that there is one such science [number theory] at any rate, and that their own, whose very remoteness from ordinary human activities should keep it gentle and clean"*

No longer: Number theory is the basis of modern public-key systems such as RSA.

Cryptography beyond communication security

Parties $1, 2, 3, \ldots, n$.
Party i has the integer $x_i \in \{0, \ldots, M-1\}$

They want to know

$$x = \frac{x_1 + \ldots + x_n}{n}$$

but each party i wants to keep its own x_i private.

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Usage:

x_i = score of student i on homework 1

x_i = vote of party i for proposition X on ballot

$\vdots$

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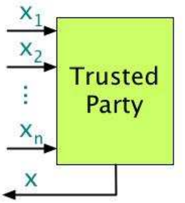

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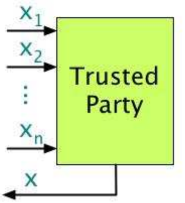

Secure Computation: Allows us to accomplish objective without a trusted party, using only (secure) communication between parties.

Internet Gambling

Will you play?

Internet Gambling

Will you play?

Casino can cheat. It returns ☹, T for some $T \neq g$

Internet Gambling

Will you play?

Casino can cheat. It returns , T for some $T \neq g$

Crypto can fix this!

Security today

- Millions of dollars of loss due to credit-card fraud, phishing, identity theft, ...
- Lack of privacy: Enormous amounts of information about each of us is collected and harvested by businesses dedicated to this purpose

Cryptography is a central tool in getting more security and privacy

Cryptography in the real world

Central uses: SSL, SSH, TLS, IPSEC, ...

Cryptography in the real world

- Poor exposition: Incomplete, unclear scheme specifications in documents
- Lack of precise goal formulations
- Complex, unclear or incorrect schemes

Lack of cryptographic education and skill in workforce.

What you can get from this course

You can get the ability to

- Identify threats
- Evaluate security solutions and technologies
- Design high-quality solutions
- Write clear, complete scheme specifications

If nothing else, develop a healthy sense of paranoia!

Administrative

Resources:

- Lecture slides
- Course notes
- Homework solutions

No textbook.

All resources on course web page.

Administrative

- Read course information sheet!
  Handout today and on course webpage.
- Grades based on
  - homeworks

  No exams, no projects.

Rules

- Collaboration with upto one other CSE207 student allowed <u>if</u> so indicated on problem set, but each student must write their own solutions in their own words.
- Looking at solutions from previous years of the course or finding them on the Internet is not allowed.

Grading

- Strive for neat, mathematically precise and well-written solutions.
- Type-setting of homeworks is encouraged, but not mandated
- Quality of exposition will impact score.

Pre-requisites

This is a theory course! Largely definitions and proofs, although of applied value.

Needed: undergraduate algorithms and theory of computation, some probability theory, a little calculus, and

MATHEMATICAL MATURITY

Warm-up

Question: What is the cost of multiplying two k-bit numbers?

Warm-up

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Answer: $O(k^2)$

$$
\begin{array}{cccccccccc}
 & & & 1 & 0 & 1 & 1 & 1 & 0 \\
 & & \times & & & & 1 & 0 & 1 \\
\hline
 & & & 1 & 0 & 1 & 1 & 1 & 0 \\
 & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
+ & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
\hline
 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0
\end{array}
$$

Warm-up

Question: I have a coin with probability p of HEADS. I flip it n times.

$$\text{Pr}[\text{at least one HEADS}] \quad =$$

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$$\Pr[\text{at least one HEADS}] = pn$$

Because I flip n coins and each has probability p of being HEADS.

Question: I have a coin with probability p of HEADS. I flip it n times.

$$\Pr[\text{at least one HEADS}] = pn$$

WRONG! Why?

Say $p = \frac{1}{2}$ and $n = 3$. Then the "probability" is

$$pn = \frac{1}{2}(3) = \frac{3}{2} > 1 \text{ ??}$$

Warm-up

Question: I have a coin with probability p of HEADS. I flip it n times.

$$\Pr[\text{at least one HEADS}] \quad = \quad pn$$

WRONG! Why?

Let H_i be the event that the i-th flip is heads.

$\Pr[H_i] = p$ for all $1 \leq i \leq n$

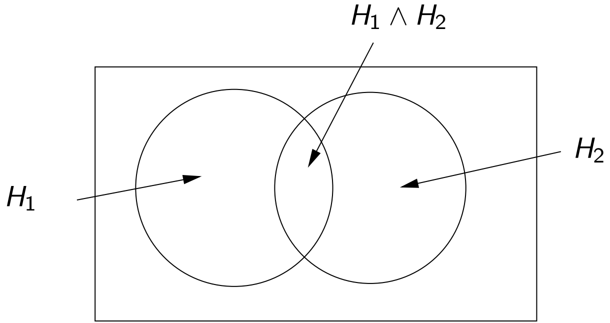
$\Pr[\text{at least one HEADs}] = \Pr[H_1 \vee H_2 \vee \cdots \vee H_n]$

but this is not equal to

$$\Pr[H_1] + \cdots + \Pr[H_n]$$

Warm-up

Example: $n = 2$



$$\Pr[H_1 \vee H_2] = \Pr[H_1] + \Pr[H_2] - \Pr[H_1 \wedge H_2]$$

Is there another way to compute

$$\Pr[\text{at least one HEADs}]\,?$$

Warm-up

Question: I have a coin with probability p of HEADS. I flip it n times.

$$\begin{aligned} \Pr[\text{ at least one HEADS}] &= 1 - \Pr\left[\text{all TAILs}\right] \\ &= 1 - (1-p)^n \end{aligned}$$