Name: _____

Entry number: _____

- Always try to give algorithm with best possible running time. The points that you obtain will depend on the running time of your algorithm. For example, a student who gives an $O(n)$ algorithm will receive more points than a student who gives an $O(n^2)$ algorithm.

- You are required to give proofs of correctness whenever needed. For example, if you give an algorithm using network flow for some problem, then you should also give a proof why this algorithm outputs optimal solution.

- **Use of unfair means will be severely penalized.**

There are 3 questions for a total of 20 points.

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(10) 1. (*Vertex cover of a Tree*) Recall, a vertex cover of a graph is a subset of vertices that includes at least one endpoint of every edge. Design an algorithm to find the size of the smallest vertex cover of a given *tree* $T = (V, E)$. Recall, a tree is a connected graph without cycles. Discuss running time of your algorithm.

2. You are given a directed graph G with positive integer capacities, a source vertex s, and a sink vertex t. You are also given a maximum s-t slow f in G. This maximum flow f has the property that there is no cycle in G on which all edges carry positive flow. Suppose we pick an edge e and reduce its capacity by 1 unit. Let us call this resulting graph G'.

(3)   (a) <u>Prove or disprove</u>: The value of maximum flow in G' is strictly less than the value of f.

(7)   (b) Design an algorithm to find a maximum flow in G'. Discuss running time.

3. (*Extra credit question* (**3 points**)) A *clique* in an undirected graph $G = (V, E)$ is a subset of vertices C such that for any two vertices $u, v \in C$, there is an edge between u and v. The CLIQUE problem is defined as follows:

   CLIQUE: Given an undirected graph $G = (V, E)$ and an integer k, determine if there is a clique of size at least k.

   Recall, an independent set in an undirected graph is a subset of vertices such that no two vertices in the subset has an edge between them. Also recall the independent set problem.

   INDEPENDENT-SET: Given an undirected graph $G = (V, E)$ and an integer k, determine if there is an independent set of size at least k.

   Show that INDEPENDENT-SET $\leq_p$ CLIQUE.

Space for rough work