Divide and Conquer (contd.)

NON-OBVIOUS APPLICATIONS OF PARALLEL SCAN / REDUCTIONS

Suppose, we have an array of 0's and 1's, and we want to determine how many 1's begin the array.

- Ex (1,1,1,0,1,1,0,1)..The answer is 3.

It may be non-intuitive to think of an associative operator which we might use here!

However, there seems to be a common trick, which we can try to learn.

THE TRICK

Let us define for any segment of the array by the notation (x,p)

- x denotes the number of leading 1's
- p denotes whether the segment contains only 1's.

Thus, each element a_i is replaced by (a_i,a_i).

How do we combine, (x,p) and (y,q)?

Let us define an operator, $\otimes$ to do this.

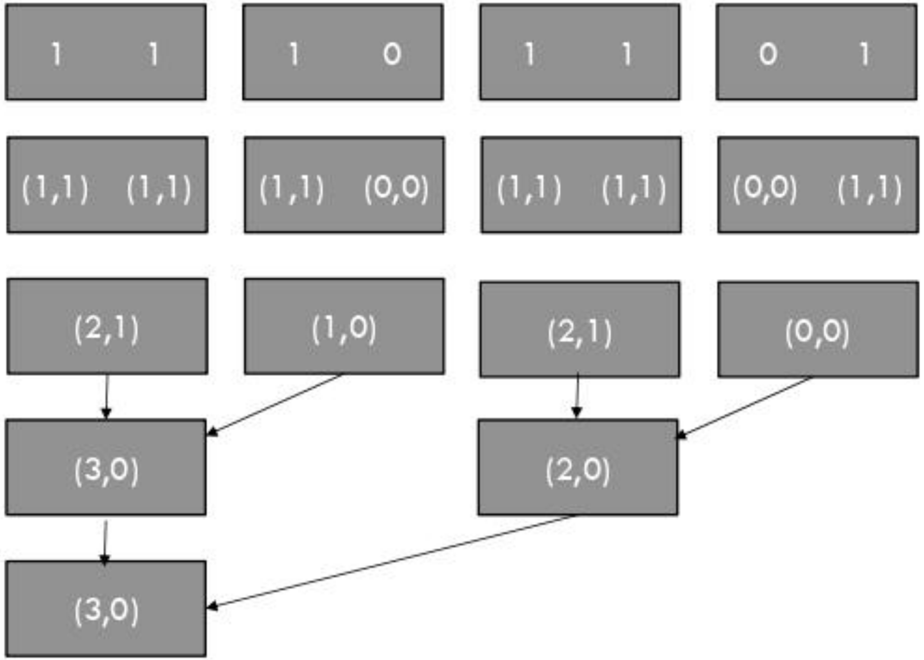
It is intuitive that $(x,p) \otimes (y,q)=(x+py,pq)$. Why?

Is this operator associate?

- $((x,p) \otimes (y,q)) \otimes (z,r)=(x+py,pq) \otimes (z,r)=(x+py+pqz,pqr)$
- $(x,p) \otimes ((y,q)) \otimes (z,r))=(x,p)\otimes(y+qz,qr)=(x+p(y+qz),pqr)=(x+py+pqz,pqr)$

Now all the previous parallelizations can be applied ☺

EXAMPLE



Evaluating polynomial: Estrin's scheme

Take $P_n(x)$ to mean the nth order polynomial of the form: $P_n(x) = C_0 + C_1x + C_2x^2 + C_3x^3 + \cdots + C_nx^n$

Written with Estrin's scheme we have:

$P_3(x) = (C_0 + C_1x) + (C_2 + C_3x)\, x^2$

$P_4(x) = (C_0 + C_1x) + (C_2 + C_3x)\, x^2 + C_4x^4$

$P_5(x) = (C_0 + C_1x) + (C_2 + C_3x)\, x^2 + (C_4 + C_5x)\, x^4$

$P_6(x) = (C_0 + C_1x) + (C_2 + C_3x)\, x^2 + ((C_4 + C_5x) + C_6x^2)x^4$

In full detail, consider the evaluation of $P_{15}(x)$:

Inputs: x, C_0, C_1, C_2, C_3, C_4, C_5 C_6, C_7, C_8, C_9 C_{10}, C_{11}, C_{12}, C_{13} C_{14}, C_{15}

Step 1: x^2, C_0+C_1x, C_2+C_3x, C_4+C_5x, C_6+C_7x, C_8+C_9x, $C_{10}+C_{11}x$, $C_{12}+C_{13}x$, $C_{14}+C_{15}x$

Step 2: x^4, $(C_0+C_1x) + (C_2+C_3x)x^2$, $(C_4+C_5x) + (C_6+C_7x)x^2$, $(C_8+C_9x) + (C_{10}+C_{11}x)x^2$, $(C_{12}+C_{13}x) + (C_{14}+C_{15}x)x^2$

Step 3: x^8, $((C_0+C_1x) + (C_2+C_3x)x^2) + ((C_4+C_5x) + (C_6+C_7x)x^2)x^4$, $((C_8+C_9x) + (C_{10}+C_{11}x)x^2) + ((C_{12}+C_{13}x) + (C_{14}+C_{15}x)x^2)x^4$

Step 4: $(((C_0+C_1x) + (C_2+C_3x)x^2) + ((C_4+C_5x) + (C_6+C_7x)x^2)x^4) + (((C_8+C_9x) + (C_{10}+C_{11}x)x^2) + ((C_{12}+C_{13}x) + (C_{14}+C_{15}x)x^2)x^4)x^8$

CAN YOU EVALUATE A POLYNOMIAL IN PARALLEL USING A SIMILAR METHOD?

Consider a polynomial : $a_0x^{n-1}+a_1x^{n-2}+\ldots+a_{n-2}x+a_{n-1}$.

Each segment also denotes a polynomial. Say, the first two coefficients denoted a_0x+a_1

Let us consider (p,y) to denote a segment.

- p denotes the value of the segment's polynomial evaluated for x
- y denotes the value of x^n, where n is the length of the segment

Thus, each element a_i is replaced by (a_i,x).

How do we combine, (p,y) and (q,z)?

Let us define an operator, $\otimes$ to do this.

It is intuitive that $(p,y) \otimes (q,z)=(pz+q,yz)$. Why?

Is this operator associate?

- $((a,x) \otimes (b,y)) \otimes (c,z)=(ay+b,xy) \otimes (c,z)=(ayz+bz+c, xyz)$
- $(a,x) \otimes ((b,y)) \otimes (c,z))=(a,x) \otimes (bz+c,yz)=(ayz+bz+c, xyz)$

Now all the previous parallelizations can be applied ☺