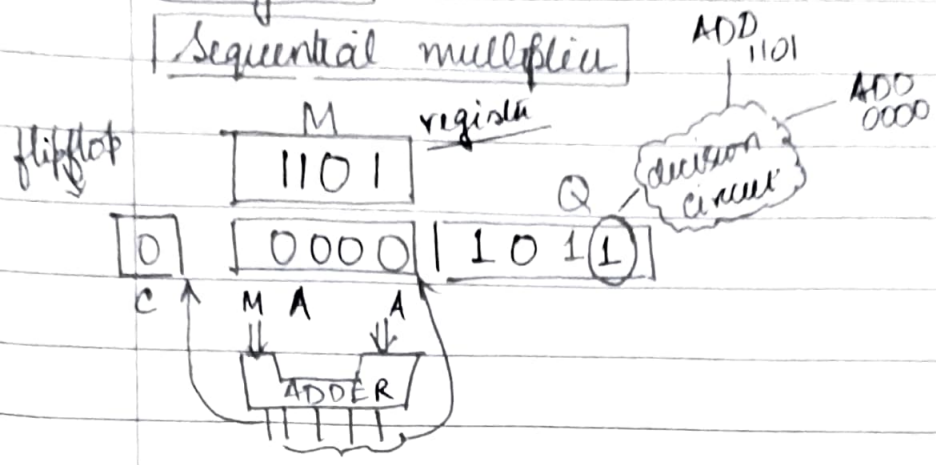


Binary multiplication

1. Unsigned

Sequential multiplier



```

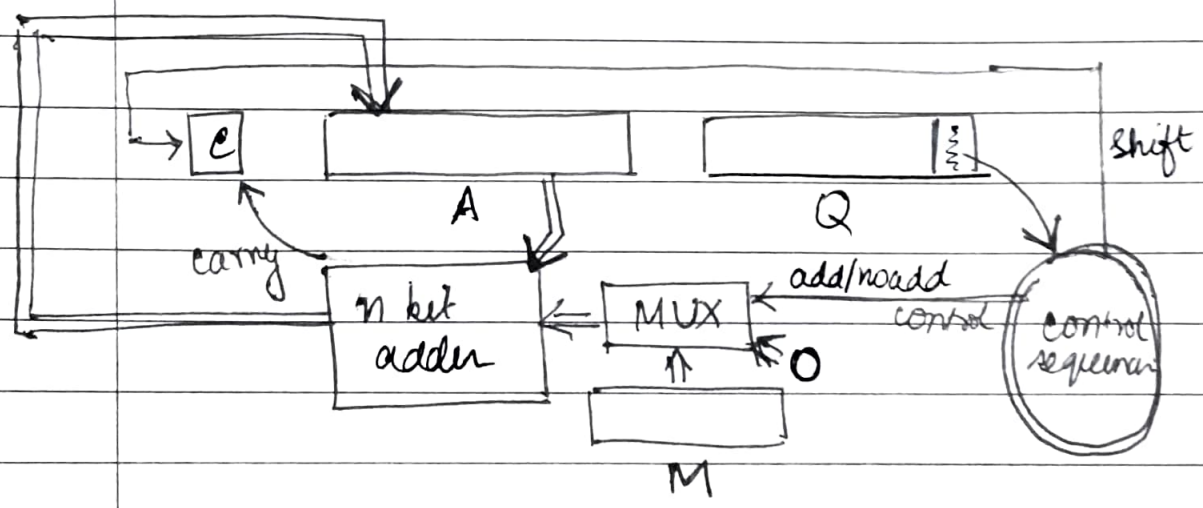
      1101 (13)
    1011 (11)
    -----
    0000
    1101
    1101
    1101
    -----
    100111 → new
    0000   add
    -----
    100111 → shift
    1101   add
    -----
    10001111
    
```

Add	0	1101	1011	1st cycle
Shift R	0	0110	1101	2nd cycle
Add	1	0011	1101	
Shift R	0	1001	1110	3rd cycle
Shift R	0	0100	1111	4th cycle
Add	1	0001	1111	
Shift R	0	1000	1111	→ result/product

```

    100111 → new
    0000   add
    -----
    100111 → shift
    1101   add
    -----
    10001111
    
```

$2^7 = 128 + 15 = 143$



(multiplicand) $\frac{11}{M} \times \frac{13}{Q}$ (multiplier)

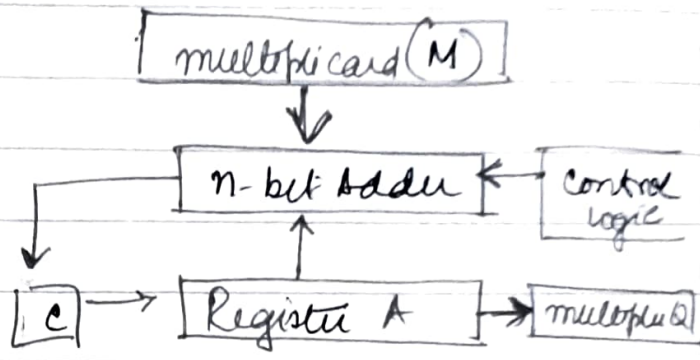
4 bit architecture
[unsigned]

Tracing table

M	C	A	Q	operation
1011	0	0000	1101	initialization
	0	1011	1101	First cycle: add $A = A + M$
	0	0101	1110	shift R CAQ
	0	0010	1111	Second cycle: shift R CAQ
	0	1101	1111	Third cycle: Add $A = A + M$
	0	0110	1111	Shift R CAQ
	1	0001	1111	Fourth cycle Add $A = A + M$
	0	1000	1111	Shift R CAQ

2^7 $2^3 2^2 2^1 2^0$
 $128 + 15 = 143$

Hardware structure



2. Signed number

multiplicand 1 1 0 1 negative $-2^3 + 2^2 + 2^0 = -8 + 4 + 1$
 multiplier 0 1 1 1 positive $[7] \text{ } [product -21] = [-3]$

~~1 1 0 1~~ (-3) ✓ sign extension
 $-2^4 + 2^3 + 2^2 + 2^0 = -16 + 8 + 4 + 1 = -3$
 should be $-3 \times 3 = -9$ but we have -3
 $1 1 0 1 1 1$ ✓ $-2^5 + 2^2 + 2^1 + 2^0 = -32 + 7 = -25$ ✗
 $1 1 0 1$
 $1 1 0 1 0 1 1$ ✓ $-2^5 + 2^4 + 2^2 + 2^1 + 2^0 = -32 + 16 + 7 = -9$ ✓
 $-64 + 32 + 8 + 2 + 1 = -64 + 43 = -21$

signed no.

non-neg. non-neg.	neg non-neg.	non-neg. neg.	neg neg.
	sign extension	take 2's complement of both	take 2's complement of both

0/1 → multiplicand
 ↻ positive/negative

3×-7
 -3×7
 $= -21$

-3×-7
 3×7
 $= 21$

original 0011 X 1001
 2's complement 1101 X 0111

Divisor (3)

Dividend (11)

Date ___/___/___

n	M	A	Q	Operation
4	00011	00000	1011	Initialization
	M' 11101	00001	011?	First cycle: shift left AQ
		①1110	011?	A = A - M
3		00001	0110	Q[0] ← 0, restore A
		00010	110?	SL AQ
		①1111	110?	A = A - M
2		00010	1100	Q[0] ← 0, restore A
		00101	100?	SL AQ
		①0010	100?	A = A - M
1		00010	1001	Q[0] ← 1
		00101	001?	SL AQ
0		00010	0010	A = A - M
		00010	0011	Q[0] ← 1

↓
↑
 Remainder Quotient

