COL865: Special Topics in Computer Applications

Physics-Based Animation

9 — Collision response

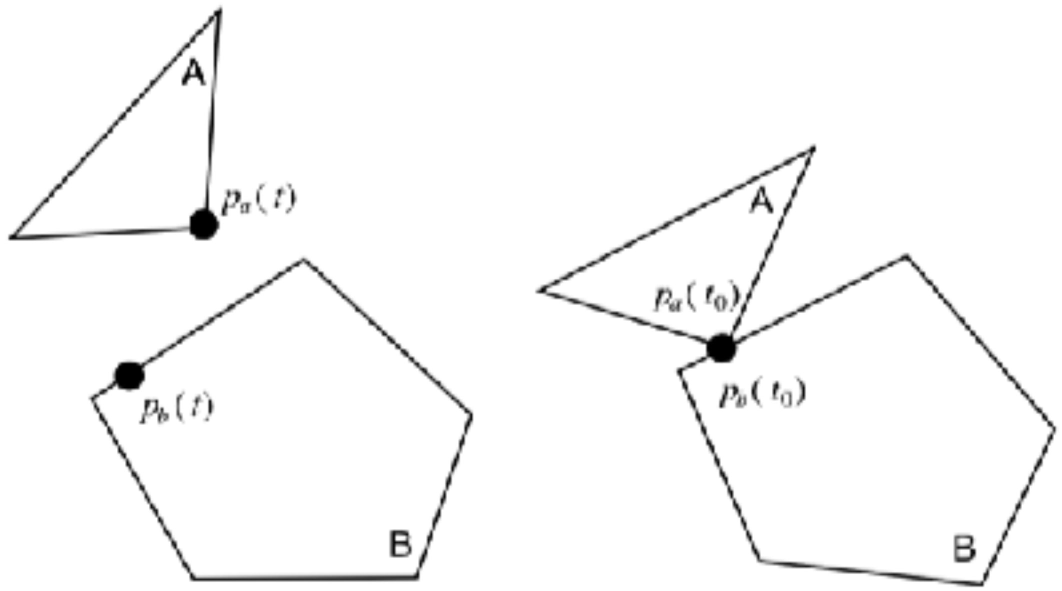
Reading

- Witkin and Baraff, *Physically Based Modeling*, Ch. “Rigid Body Simulation” Sec. 6, 8, 9
- Guendelman et al., “Nonconvex Rigid Bodies with Stacking”, 2003

Collision detection

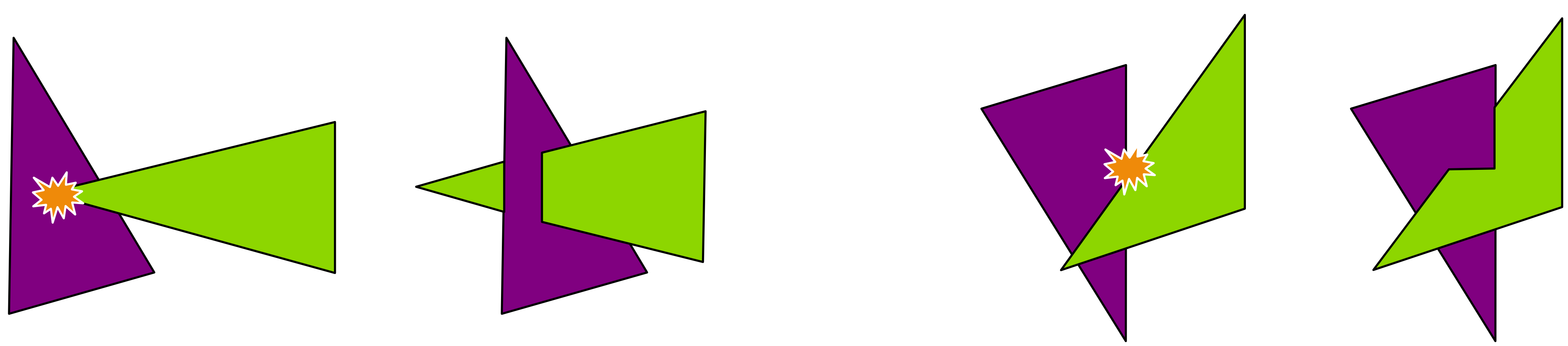
Collision detection routine returns **contact pairs**:

- Point $\mathbf{p}_1$ on body 1
- Point $\mathbf{p}_2$ on body 2
- Collision normal $\mathbf{n}$



[Witkin & Baraff]

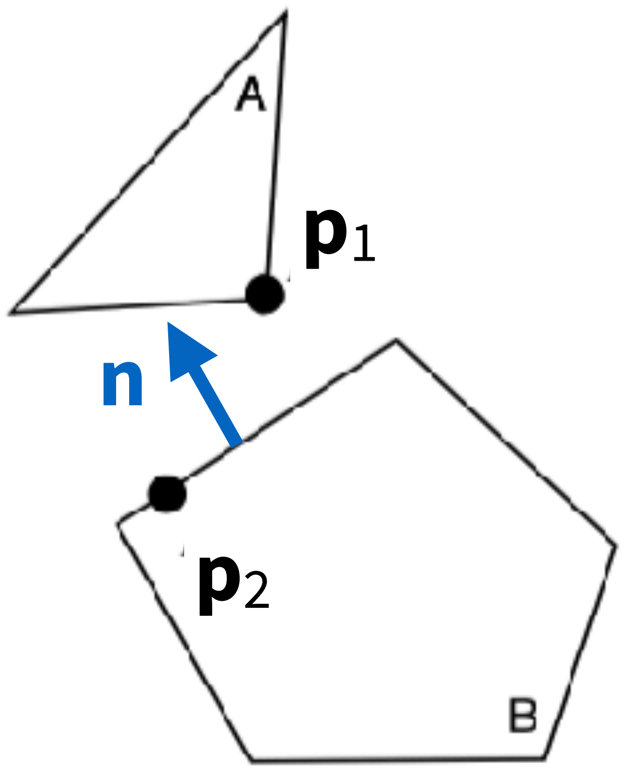
In 3D, collision can be vertex-face or edge-edge



Nonpenetration constraints

Define constraint function $g \geq 0$:

$$\mathbf{n} \cdot (\mathbf{p}_1 - \mathbf{p}_2) \geq 0$$

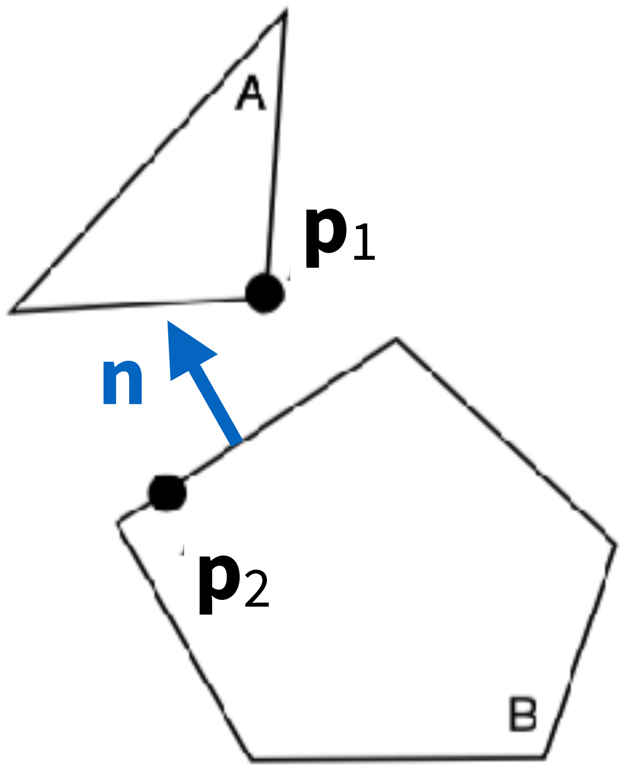


$\mathbf{n}$ =

- face normal (if vertex-face collision)
- cross product of edges (if edge-edge)

A single collision

$$g = \mathbf{n} \cdot (\mathbf{p}_1 - \mathbf{p}_2) \geq 0$$



At time of collision, $\mathbf{p}_1 - \mathbf{p}_2 = \mathbf{0}$

Velocity constraint:

$$\dot{g} = \mathbf{J}\,\mathbf{u} \geq 0$$

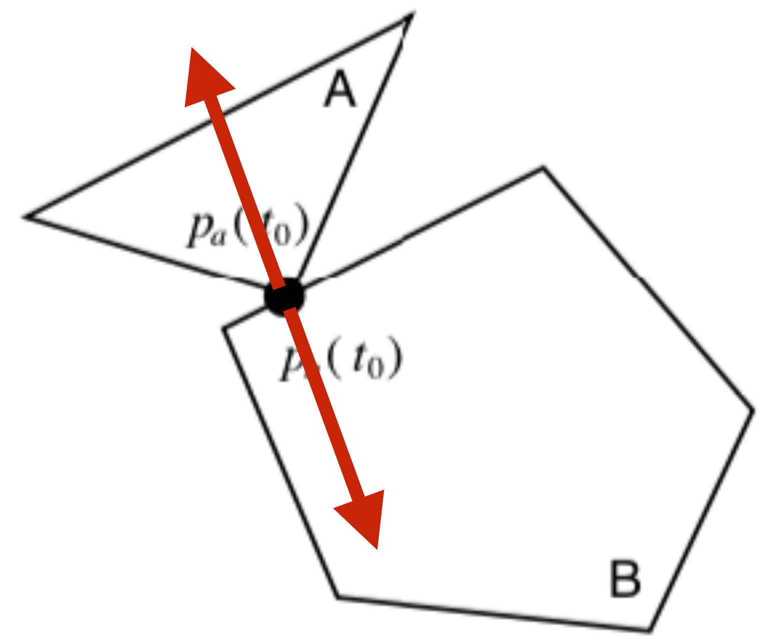
$$v_{\text{rel,n}} = \mathbf{n} \cdot (\dot{\mathbf{p}}_1 - \dot{\mathbf{p}}_2) \geq 0$$

$$\begin{bmatrix} \mathbf{n}^T & (\mathbf{r}_1 \times \mathbf{n})^T & -\mathbf{n}^T & -(\mathbf{r}_2 \times \mathbf{n})^T \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \boldsymbol{\omega}_1 \\ \mathbf{v}_2 \\ \boldsymbol{\omega}_2 \end{bmatrix} \geq 0$$

Collision impulse

Normal force of magnitude f_n produces forces and torques $[\mathbf{f}_1 ; \boldsymbol{\tau}_1; \mathbf{f}_2 ; \boldsymbol{\tau}_2] = \mathbf{J}^T f_n$

Collisions require normal ***impulse*** j_n which instantaneously changes velocity



$$v_n^+ = v_n^- + \mathbf{J}\,\mathbf{M}^{-1}\,\mathbf{J}^T j_n$$

where

$$\mathbf{M} = \begin{bmatrix} m_1\mathbf{1} & & & \\ & \mathbf{I}_1 & & \\ & & m_2\mathbf{1} & \\ & & & \mathbf{I}_2 \end{bmatrix}$$

Exercise: Verify that this is equivalent to applying equal and opposite normal forces $\pm f_n\,\mathbf{n}$ to both bodies.

Collision impulse

- If $v_n^- > 0$, bodies are already separating. Set $j_n = 0$
- If $v_n^- = 0$, sliding or resting contact
- If $v_n^- < 0$, collision! Compute j_n to push bodies away

$$v_n^+ = v_n^- + \mathbf{J}\,\mathbf{M}^{-1}\,\mathbf{J}^T\,j_n$$

What should v_n^+ be?

Newton's hypothesis: $v_n^+ = -\varepsilon\, v_n^-$, where $0 \le \varepsilon \le 1$

Deformable body collisions

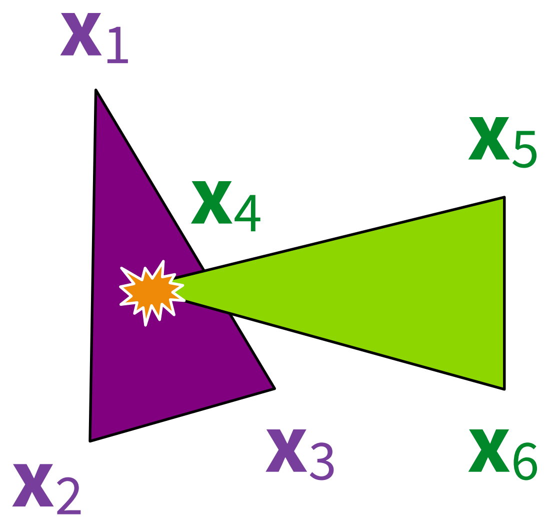
Same applies to deformable bodies!

Take a mass-spring system with springs along triangle edges:

- $\mathbf{p}_1 = b_1\,\mathbf{x}_1 + b_2\,\mathbf{x}_2 + b_3\,\mathbf{x}_3$
- $\mathbf{p}_2 = \mathbf{x}_4$

What are $\mathbf{J}$ and $\mathbf{u}$ here?

What is $\mathbf{J}\,\mathbf{M}^{-1}\,\mathbf{J}^{T}$?



Friction

Friction force is tangential, magnitude depends on f_{n}

Coulomb model:

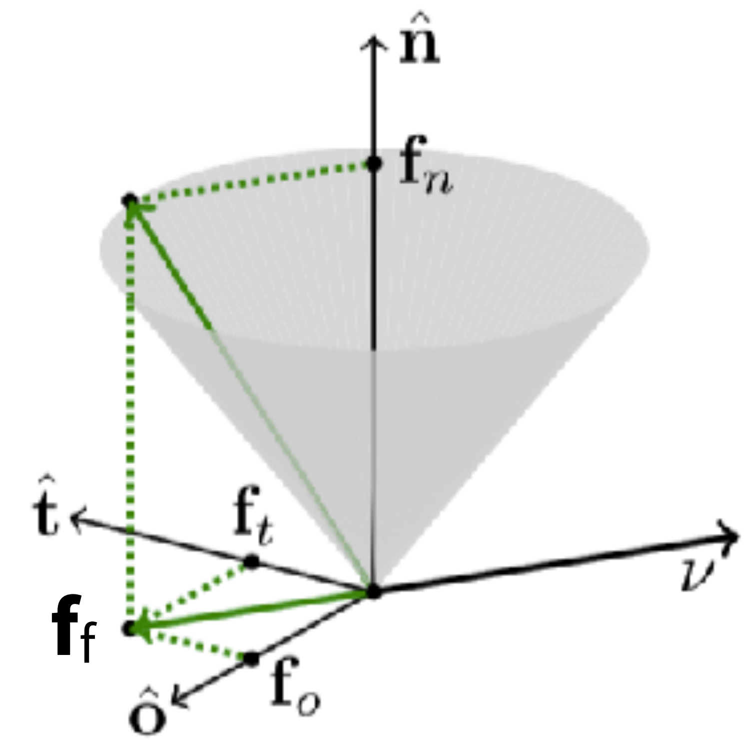
$$\|\mathbf{f}_{\mathrm{f}}\| \leq \mu\, f_{\mathrm{n}}$$

Maximum dissipation principle:

$$\max -\dot{T} \quad \text{s.t.} \quad \|\mathbf{f}_{\mathrm{f}}\| \leq \mu\, f_{\mathrm{n}}$$

$$\Rightarrow \mathbf{f}_{\mathrm{f}} \parallel -\mathbf{v}_{\mathrm{rel,t}}$$

For impulses, min T^{+}

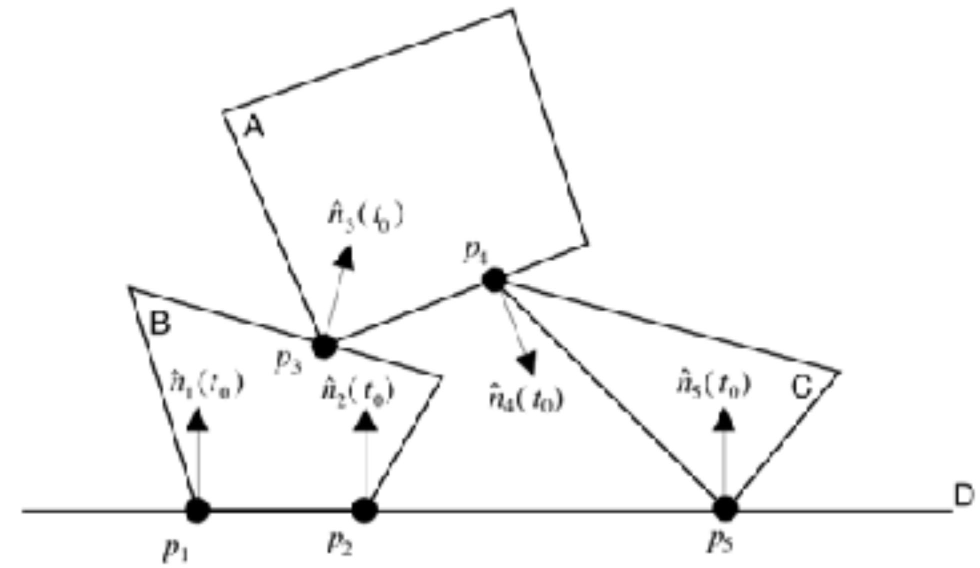


[Bender et al. 2012]

Multiple contacts

Multiple contacts

Impulse for one contact affects constraints of other contacts

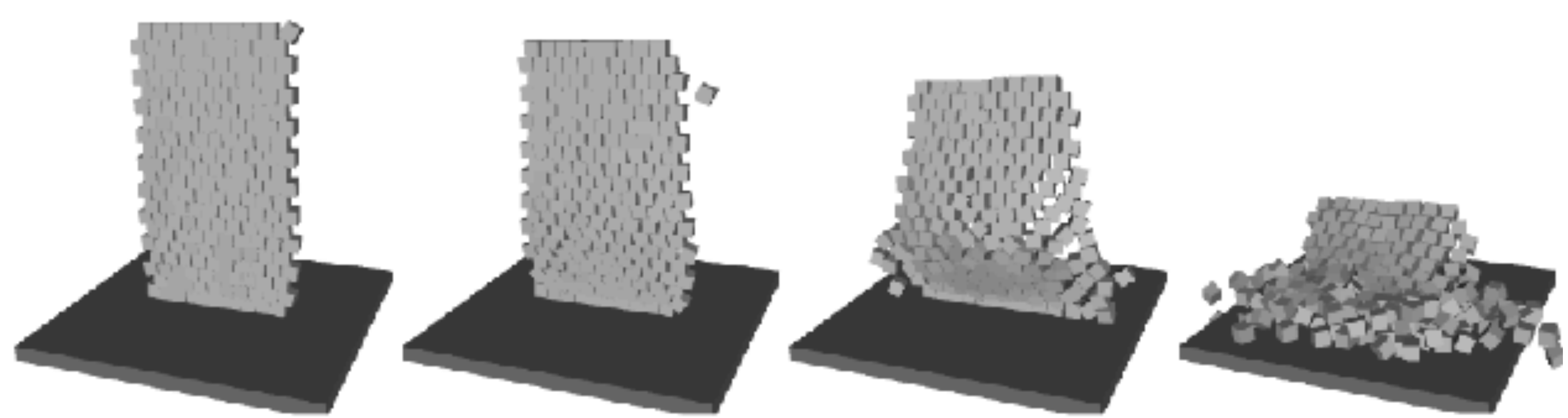


[Witkin & Baraff]

Projected Gauss-Seidel:
solve one contact at a time

- Solve $j_{i,\mathrm{n}}$ for $v_{i,\mathrm{n}}^{+} = -\varepsilon\, v_{i,\mathrm{n}}^{-}$, project to nearest $j_{i,\mathrm{n}} \geq 0$
- Solve $\mathbf{j}_{i,\mathrm{f}}$ for $\mathbf{v}_{i,\mathrm{t}} = 0$, project to nearest $\|\mathbf{j}_{i,\mathrm{f}}\| \leq \mu\, j_{i,\mathrm{n}}$

Slow to converge
for large stacks!

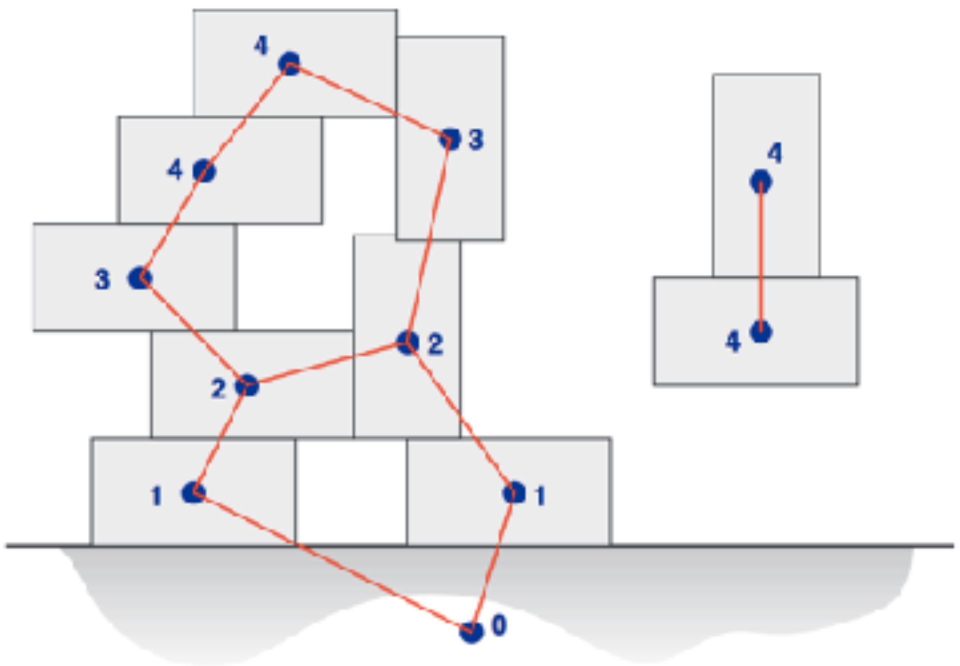


[Erleben 2004]

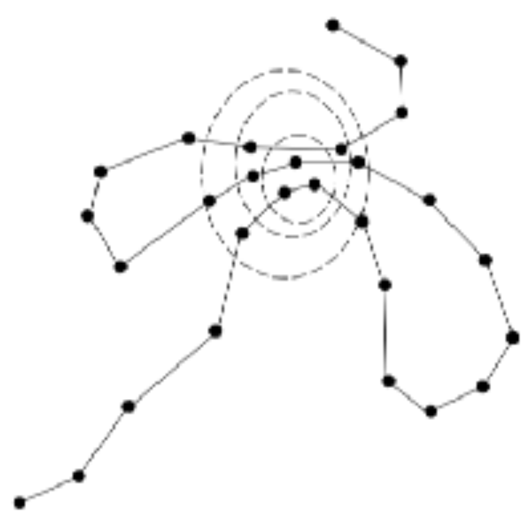
Fixing convergence failure

Freezing methods:

- ***Shock propagation*** for rigid bodies [Guendelman et al. 2003, Erleben 2007]
- ***Impact zones*** for cloth [Provot 1997, Bridson et al. 2003, Harmon et al. 2008]



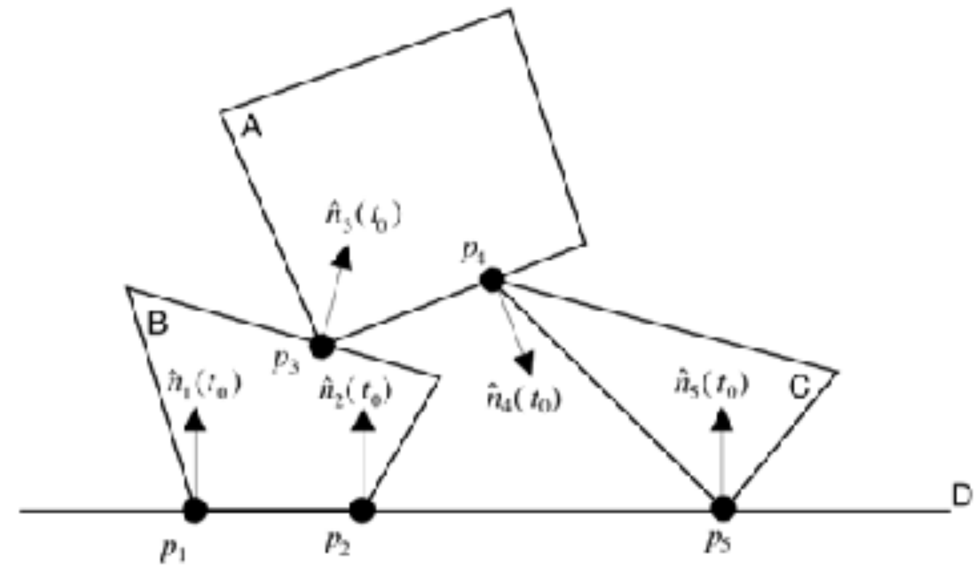
[Erleben 2005]



[Provot 1997]

Multiple contacts

Impulse for one contact affects constraints of other contacts



[Witkin & Baraff]

Complementarity:

$$0 \le v_{i,\mathrm{n}}^{+} \perp j_{i,\mathrm{n}} \ge 0$$

System of complementarity conditions for impulses $j_{i,\mathrm{n}}$, $\mathbf{j}_{i,\mathrm{f}}$

$$0 \le v_{1,\mathrm{n}}^{+} \perp j_{1,\mathrm{n}} \ge 0$$
$$0 \le \|\mathbf{v}_{1,\mathrm{t}}^{+}\| \text{ “}\perp\text{” } \|\mathbf{j}_{1,\mathrm{f}}\| \le \mu\, j_{1,\mathrm{n}}$$
$$0 \le v_{2,\mathrm{n}}^{+} \perp j_{2,\mathrm{n}} \ge 0$$
$$0 \le \|\mathbf{v}_{2,\mathrm{t}}^{+}\| \text{ “}\perp\text{” } \|\mathbf{j}_{2,\mathrm{f}}\| \le \mu\, j_{2,\mathrm{n}}$$
$$\vdots$$

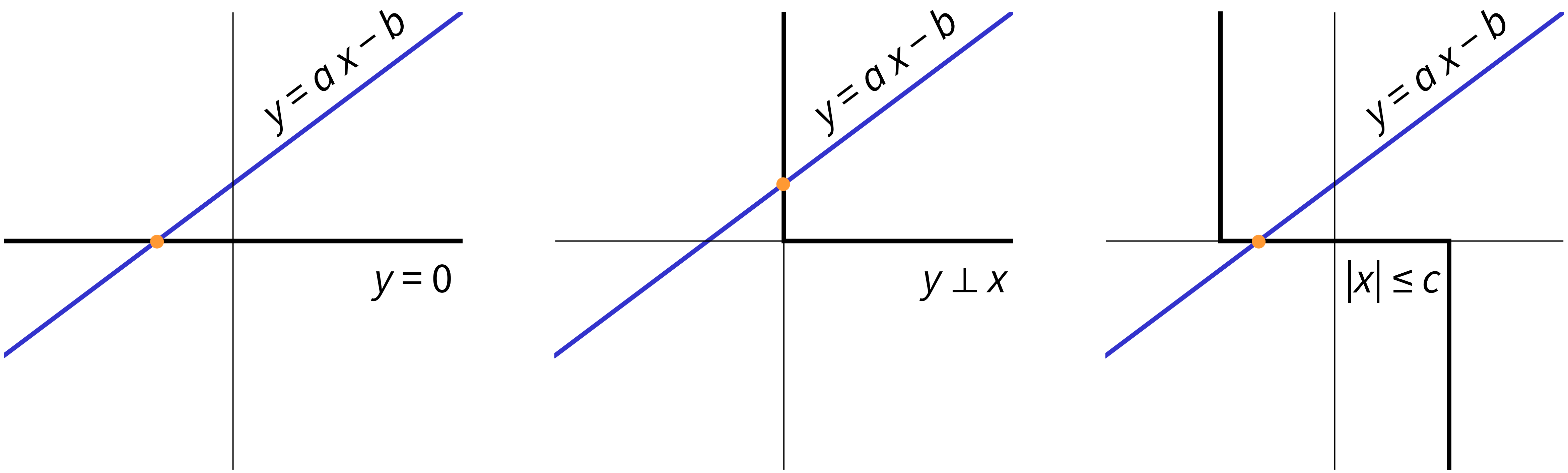
Complementarity

Linear system:

$$\mathbf{A}\,\mathbf{x} - \mathbf{b} = \mathbf{0}$$

Linear complementarity problem:

$$\mathbf{0} \le \mathbf{A}\,\mathbf{x} - \mathbf{b} \;\perp\; \mathbf{x} \ge \mathbf{0}$$



Next week

- ***This Saturday***: no class
- ***Monday***: Collision detection (guest lecture by Prof. Kalra)
- ~~***Wednesday***~~: Independence Day
- ***Thursday***: Paper discussions
  [Provot 1997, Weinstein et al. 2006]
  - Assignment 1 due at midnight!