# Fundamentals of Decision Theory
## Chapter 16

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(Based on slides of someone from NPS, Maria Fasli)

# Decision Theory

- **“an analytic and systematic approach to the study of decision making”**

Good decisions:

- based on reasoning
- consider all available data and possible alternatives
- employ a quantitative approach

Bad decisions:

- not based on reasoning
- do not consider all available data and possible alternatives
- do not employ a quantitative approach

  - **A good decision may occasionally result in an unexpected outcome; it is still a good decision if made properly**
  - **A bad decision may occasionally result in a good outcome if you are lucky; it is still a bad decision**

# Steps in Decision Theory

1. List the possible alternatives (actions/decisions)

2. Identify the possible outcomes

3. List the payoff or profit or reward

4. Select one of the decision theory models

5. Apply the model and make your decision

# Example
# The Thompson Lumber Company

- Problem.
  - The Thompson Lumber Co. must decide whether or not to expand its product line by manufacturing and marketing a new product, backyard storage sheds

- Step 1: List the possible alternatives

  *alternative*: "a course of action or strategy that may be chosen by the decision maker"

  - (1) Construct a large plant to manufacture the sheds
  - (2) Construct a small plant
  - (3) Do nothing

# The Thompson Lumber Company

- Step 2: Identify the states of nature
  - (1) The market for storage sheds could be favorable
    - high demand
  - (2) The market for storage sheds could be unfavorable
    - low demand

*state of nature*: "an outcome over which the decision maker has little or no control"

e.g., lottery, coin-toss, whether it will rain today

# The Thompson Lumber Company

- Step 3: List the possible rewards
  - A reward for all possible combinations of alternatives and states of nature
  - *Conditional values*: "reward depends upon the alternative and the state of nature"
    - with a favorable market:
      - a large plant produces a net profit of $200,000
      - a small plant produces a net profit of $100,000
      - no plant produces a net profit of $0
    - with an unfavorable market:
      - a large plant produces a net loss of $180,000
      - a small plant produces a net loss of $20,000
      - no plant produces a net profit of $0

# Reward tables

- A means of organizing a decision situation, including the rewards from different situations given the possible states of nature

| Actions | States of Nature | |
|---|---|---|
| | a | b |
| 1 | Reward 1a | Reward 1b |
| 2 | Reward 2a | Reward 2b |

# The Thompson Lumber Company

| | States of Nature | |
|---|---|---|
| **Actions** | | |
| | | |
| | | |
| | | |

# The Thompson Lumber Company

| | States of Nature | |
|---|---|---|
| **Actions** | **Favorable Market** | **Unfavorable Market** |
| **Large plant** | **$200,000** | **-$180,000** |
| **Small plant** | **$100,000** | **-$20,000** |
| **No plant** | **$0** | **$0** |

# The Thompson Lumber Company

- Steps 4/5: Select an appropriate model and apply it
  - Model selection depends on the operating environment and degree of uncertainty

# Future Uncertainty

- Nondeterministic

- Probabilistic

# Non-deterministic Uncertainty

| | States of Nature | |
|---|---|---|
| **Actions** | **Favorable Market** | **Unfavorable Market** |
| **Large plant** | **$200,000** | **-$180,000** |
| **Small plant** | **$100,000** | **-$20,000** |
| **No plant** | **$0** | **$0** |

- What should we do?

# Maximax Criterion

"Go for the Gold"

- Select the decision that results in the maximum of the maximum rewards
- A very optimistic decision criterion
  - Decision maker assumes that the most favorable state of nature for each action will occur
- Most risk prone agent

# Maximax

| Decision | States of Nature | | Maximum in Row |
|---|---|---|---|
| | Favorable | Unfavorable | |
| Large plant | $200,000 | -$180,000 | $200,000 |
| Small plant | $100,000 | -$20,000 | $100,000 |
| No plant | $0 | $0 | $0 |

- **Thompson Lumber Co. assumes that the most favorable state of nature occurs for each decision alternative**
- **Select the maximum reward for each decision**
  - **All three maximums occur if a favorable economy prevails (a tie in case of no plant)**
- **Select the maximum of the maximums**
  - **Maximum is $200,000; corresponding decision is to build the large plant**
  - **Potential loss of $180,000 is completely ignored**

# Maximin Criterion

"Best of the Worst"

- Select the decision that results in the maximum of the minimum rewards

- A very pessimistic decision criterion
  - Decision maker assumes that the minimum reward occurs for each decision alternative
  - Select the maximum of these minimum rewards

- Most risk averse agent

# Maximin

| Decision | States of Nature | | Minimum in Row |
|---|---|---|---|
| | Favorable | Unfavorable | |
| Large plant | $200,000 | -$180,000 | -$180,000 |
| Small plant | $100,000 | -$20,000 | -$20,000 |
| No plant | $0 | $0 | $0 |

- **Thompson Lumber Co. assumes that the least favorable state of nature occurs for each decision alternative**
- **Select the minimum reward for each decision**
  - **All three minimums occur if an unfavorable economy prevails (a tie in case of no plant)**
- **Select the maximum of the minimums**
  - **Maximum is $0; corresponding decision is to do nothing**
  - **A conservative decision; largest possible gain, $0, is much less than maximax**

# Equal Likelihood Criterion

- Assumes that all states of nature are equally likely to occur
  - Maximax criterion assumed the most favorable state of nature occurs for each decision
  - Maximin criterion assumed the least favorable state of nature occurs for each decision

- Calculate the *average reward* for each alternative and select the alternative with the maximum number
  - *Average reward*: the sum of all rewards divided by the number of states of nature

- Select the decision that gives the highest average reward

# Equal Likelihood

| Decision | States of Nature | | Row Average |
|---|---|---|---|
| | Favorable | Unfavorable | |
| Large plant | $200,000 | -$180,000 | $10,000 |
| Small plant | $100,000 | -$20,000 | $40,000 |
| No plant | $0 | $0 | $0 |

**Row Averages**

$$\text{Large Plant} = \frac{\$200{,}000 - \$180{,}000}{2} = \$10{,}000$$

$$\text{Small Plant} = \frac{\$100{,}000 - \$20{,}000}{2} = \$40{,}000$$

$$\text{Do Nothing} = \frac{\$0 + \$0}{2} = \$0$$

- Select the decision with the highest weighted value
  - Maximum is $40,000; corresponding decision is to build the small plant

# Criterion of Realism

- Also known as the weighted average or Hurwicz criterion
  - A compromise between an optimistic and pessimistic decision

- A coefficient of realism, $\alpha$, is selected by the decision maker to indicate optimism or pessimism about the future

$$0 \leq \alpha \leq 1$$

When $\alpha$ is close to 1, the decision maker is optimistic.

When $\alpha$ is close to 0, the decision maker is pessimistic.

- *Criterion of realism* = $\alpha$(row maximum) + (1-$\alpha$)(row minimum)
  - A weighted average where maximum and minimum rewards are weighted by $\alpha$ and (1 - $\alpha$) respectively

# Criterion of Realism

- Assume a coefficient of realism equal to 0.8

| Decision | States of Nature | | Criterion of Realism |
| --- | --- | --- | --- |
| | Favorable | Unfavorable | |
| Large plant | $200,000 | -$180,000 | $124,000 |
| Small plant | $100,000 | -$20,000 | $76,000 |
| No plant | $0 | $0 | $0 |

Weighted Averages

Large Plant = (0.8)($200,000) + (0.2)(-$180,000) = $124,000

Small Plant = **(0.8)($100,000) + (0.2)(-$20,000) = $76,000**

Do Nothing = **(0.8)($0) + (0.2)($0) = $0**

Select the decision with the highest weighted value

**Maximum is $124,000; corresponding decision is to build the large plant**

# Minimax Regret

- Regret/Opportunity Loss: “the difference between the optimal reward and the actual reward received”

- Choose the alternative that <u>minimizes the maximum regret</u> associated with each alternative

  – Start by determining the maximum regret for each alternative

  – Pick the alternative with the minimum number

# Regret Table

- If I knew the future, how much I'd regret my decision…

- Regret for any state of nature is calculated by subtracting each outcome in the column from the best outcome in the same column

# Minimax Regret

| Decision | States of Nature | | | | |
|---|---|---|---|---|---|
| | Favorable | | Unfavorable | | Row |
| | Payoff | Regret | Payoff | Regret | Maximum |
| Large plant | $200,000 | $0 | -$180,000 | $180,000 | $180,000 |
| Small plant | $100,000 | $100,000 | -$20,000 | $20,000 | $100,000 |
| No plant | $0 | $200,000 | $0 | $0 | $200,000 |
| Best payoff | $200,000 | | $0 | | |

- Select the alternative with the lowest maximum regret

**Minimum is $100,000; corresponding decision is to build a small plant**

# Summary of Results

| Criterion | Decision |
|---|---|
| Maximax | Build a large plant |
| Maximin | Do nothing |
| Equal likelihood | Build a small plant |
| Realism | Build a large plant |
| Minimax regret | Build a small plant |

# Future Uncertainty

- Non deterministic
- Probabilistic

# Probabilistic Uncertainty

- Decision makers know the probability of occurrence for each possible outcome
  - Attempt to maximize the expected reward

- Criteria for decision models in this environment:
  - <u>Maximization</u> of expected reward
  - <u>Minimization</u> of expected regret
    - Minimize expected regret = maximizing expected reward!

# Expected Reward (Q)

- called Expected Monetary Value (EMV) in DT literature
- "the probability weighted sum of possible rewards for each alternative"
  - Requires a reward table with conditional rewards and probability assessments for all states of nature

Q(action a) = (reward of 1st state of nature)

X (probability of 1st state of nature)

+ (reward of 2nd state of nature)

X (probability of 2nd state of nature)

+ . . . + (reward of last state of nature)

X (probability of last state of nature)

# The Thompson Lumber Company

- Suppose that the probability of a favorable market is exactly the same as the probability of an unfavorable market. Which alternative would give the greatest Q?

| | States of Nature | | |
|---|---|---|---|
| | Favorable Mkt | Unfavorable Mkt | |
| **Decision** | **p = 0.5** | **p = 0.5** | **EMV** |
| **Large plant** | **$200,000** | **-$180,000** | **$10,000** |
| **Small plant** | **$100,000** | **-$20,000** | **$40,000** |
| **No plant** | **$0** | **$0** | **$0** |

Q(large plant) = **(0.5)($200,000) + (0.5)(-$180,000) = $10,000**

Q(small plant) = **(0.5)($100,000) + (0.5)(-$-20,000) = $40,000**

Q(no plant) = **(0.5)($0) + (0.5)($0) = $0**

**Build the small plant**

# Expected Value of Perfect Information (EVPI)

- It may be possible to purchase additional information about future events and thus make a better decision
  - Thompson Lumber Co. could hire an economist to analyze the economy in order to more accurately determine which economic condition will occur in the future
    - How valuable would this information be?

# EVPI Computation

- Look first at the decisions under each state of nature
  - If information was available that perfectly predicted which state of nature was going to occur, the best decision for that state of nature could be made
    - *expected value with perfect information* (EV w/ PI): "the expected or average return if we have perfect information before a decision has to be made"

# EVPI Computation

- Perfect information changes environment from decision making under risk to decision making with certainty
  - Build the large plant if you know for sure that a favorable market will prevail
  - Do nothing if you know for sure that an unfavorable market will prevail

| | States of Nature | |
|---|---|---|
| **Decision** | **Favorable p = 0.5** | **Unfavorable p = 0.5** |
| **Large plant** | **$200,000** | **-$180,000** |
| **Small plant** | **$100,000** | **-$20,000** |
| **No plant** | **$0** | **$0** |

# EVPI Computation

- Even though perfect information enables Thompson Lumber Co. to make the correct investment decision, each state of nature occurs only a certain portion of the time
  - A favorable market occurs 50% of the time and an unfavorable market occurs 50% of the time
  - EV w/ PI calculated by choosing the best alternative for each state of nature and multiplying its reward times the probability of occurrence of the state of nature

# EVPI Computation

**EV w/ PI =** **(best reward for 1st state of nature)**
**X (probability of 1st state of nature)**
**+ (best reward for 2nd state of nature)**
**X (probability of 2nd state of nature)**

**EV w/ PI =** **(\$200,000)(0.5) + (\$0)(0.5) = \$100,000**

| Decision | States of Nature | |
|---|---|---|
| | Favorable p = 0.5 | Unfavorable p = 0.5 |
| Large plant | \$200,000 | -\$180,000 |
| Small plant | \$100,000 | -\$20,000 |
| No plant | \$0 | \$0 |

# EVPI Computation

- Thompson Lumber Co. would be foolish to pay more for this information than the extra profit that would be gained from having it
  - *EVPI*: "the maximum amount a decision maker would pay for additional information resulting in a decision better than one made *without perfect information* "
    - EVPI is the expected outcome with perfect information minus the expected outcome without perfect information

EVPI = EV w/ PI - Q

| |
|---|
| EVPI = $100,000 - $40,000 = $60,000 |

# Using EVPI

- EVPI of $60,000 is the maximum amount that Thompson Lumber Co. should pay to purchase perfect information from a source such as an economist
  - "Perfect" information is extremely rare
  - An investor typically would be willing to pay some amount less than $60,000, depending on how reliable the information is perceived to be

# Is Expected Value sufficient?

- Lottery 1
  - returns \$0 always
- Lottery 2
  - return \$100 and -\$100 with prob 0.5

- Which is better?

# Is Expected Value sufficient?

- Lottery 1
  - returns $100 always
- Lottery 2
  - return $10000 (prob 0.01) and $0 with prob 0.99

- Which is better?
  - depends

# Is Expected Value sufficient?

- Lottery 1
  - returns $3125 always
- Lottery 2
  - return $4000 (prob 0.75) and -$500 with prob 0.25

- Which is better?

# Is Expected Value sufficient?

- Lottery 1
  - returns $0 always
- Lottery 2
  - return $1,000,000 (prob 0.5) and -$1,000,000 with prob 0.5

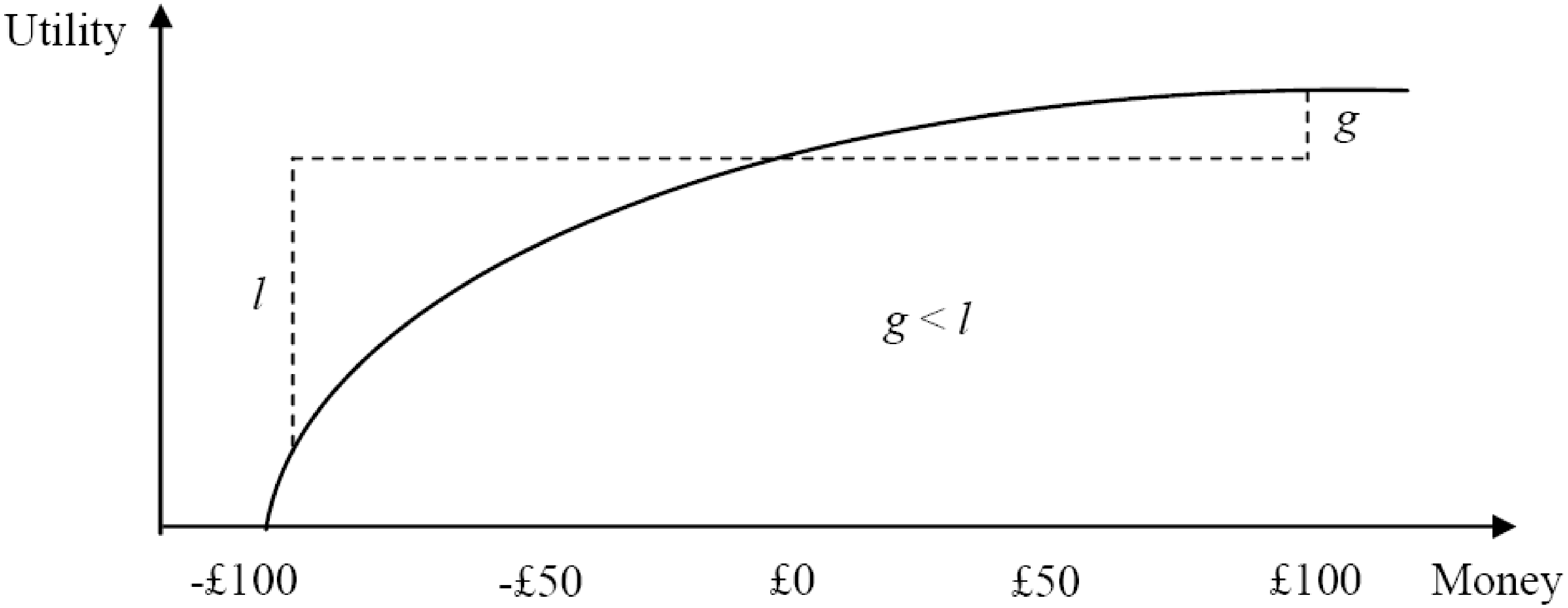
- Which is better?

# Utility Theory

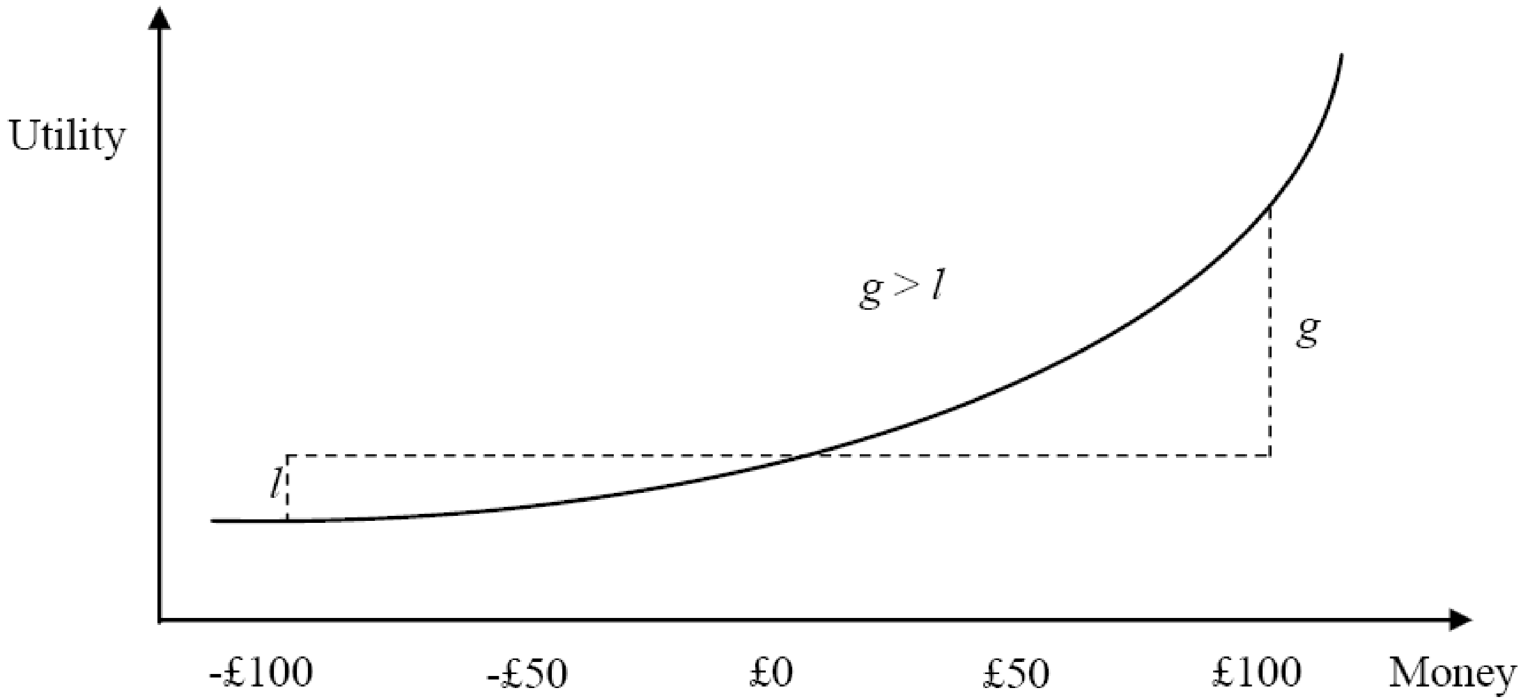
- Adds a layer of utility over rewards
- Risk averse
  - |Utility| of high negative money is much MORE than utility of high positive money
- Risk prone
  - Reverse

- Use expected utility criteria…

# Utility function of risk-averse agent



# Utility function of a risk-prone agent



# Utility function of a risk-neutral agent