# Log Linear Models for Text Classification

Mausam

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# Introduction

- So far we've looked at "generative models"
  - Naive Bayes
- But there is now much use of conditional or discriminative probabilistic models in NLP, Speech, IR (and ML generally)
- Because:
  - They give high accuracy performance
  - They make it easy to incorporate lots of linguistically important features
  - They allow automatic building of language independent, retargetable NLP modules

# Joint vs. Conditional Models

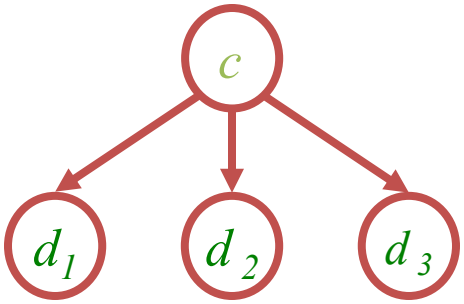
- We have some data {(*d*, *c*)} of paired observations *d* and hidden classes *c*.
- Joint (generative) models place probabilities over both observed data and the hidden stuff (generate the observed data from hidden stuff):
  - All the classic Stat-NLP models:
    - *n*-gram models, Naive Bayes classifiers, hidden Markov models, probabilistic context-free grammars, IBM machine translation alignment models

# Joint vs. Conditional Models

- Discriminative (conditional) models take the data as given, and put a probability over hidden structure given the data:
  - Logistic regression, conditional loglinear or maximum entropy models, conditional random fields
  - Also, SVMs, (averaged) perceptron, etc. are discriminative classifiers (but not directly probabilistic)

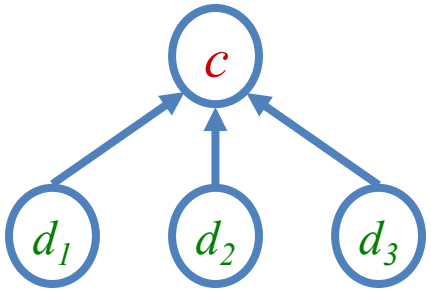
# Bayes Net/Graphical Models

- Bayes net diagrams draw circles for random variables, and lines for direct dependencies
- Some variables are observed; some are hidden
- Each node is a little classifier (conditional probability table) based on incoming arcs




Naive Bayes

Generative




Logistic Regression

Discriminative

# Conditional vs. Joint Likelihood

- A *joint* model gives probabilities P(*d*,*c*) and tries to maximize this joint likelihood.
  - It turns out to be trivial to choose weights: just relative frequencies.
- A *conditional* model gives probabilities P(*c*|*d*). It takes the data as given and models only the conditional probability of the class.
  - We seek to maximize conditional likelihood.
  - Harder to do (as we'll see…)
  - More closely related to classification error.

# Text Categorization with Word Features

| Data<br>BUSINESS: Stocks hit a yearly low … |
| --- |
| Label: BUSINESS<br>Features<br>{…, stocks, hit, a, yearly, low, …} |

(Zhang and Oles 2001)

- Features are presence of each word in a document and the document class (they do feature selection to use reliable indicator words)

- Tests on classic Reuters data set (and others)
  - Naïve Bayes: 77.0% $F_1$
  - Logistic regression: 86.4%
  - Support vector machine: 86.5%

# Case Study: Word Senses

- Words have multiple distinct meanings, or senses:
  - Plant: living plant, manufacturing plant, …
  - Title: name of a work, ownership document, form of address, material at the start of a film, …
- Many levels of sense distinctions
  - Homonymy: totally unrelated meanings (river bank, money bank)
  - Polysemy: related meanings (star in sky, star on tv)
  - Systematic polysemy: productive meaning extensions (metonymy such as organizations to their buildings) or metaphor
  - Sense distinctions can be extremely subtle (or not)
- Granularity of senses needed depends a lot on the task
- Why is it important to model word senses?
  - Translation, parsing, information retrieval?

# Word Sense Disambiguation

- Example: living plant vs. manufacturing plant
- How do we tell these senses apart?
  - "context"

    The manufacturing plant which had previously sustained the town's economy shut down after an extended labor strike.

  - Maybe it's just text categorization
  - Each word sense represents a class
  - Run a naive-bayes classifier?
- Bag-of-words classification works OK for noun senses
  - 90% on classic, shockingly easy examples (line, interest, star)
  - 80% on senseval-1 nouns
  - 70% on senseval-1 verbs

# Verb WSD

- Why are verbs harder?
  - Verbal senses less topical
  - More sensitive to structure, argument choice

- Verb Example: “Serve”
  - [function] The tree stump serves as a table
  - [enable] The scandal served to increase his popularity
  - [dish] We serve meals for the homeless
  - [enlist] She served her country
  - [jail] He served six years for embezzlement
  - [tennis] It was Agassi's turn to serve
  - [legal] He was served by the sheriff

# Better Features

- There are smarter features:
  - Argument selectional preference:
    - serve NP[meals] vs. serve NP[papers] vs. serve NP[country]
  - Subcategorization:
    - [function] serve PP[as]
    - [enable] serve VP[to]
    - [tennis] serve <intransitive>
    - [food] serve NP {PP[to]}

- Other constraints (Yarowsky 95)
  - One-sense-per-discourse (only true for broad topical distinctions)
  - One-sense-per-collocation (pretty reliable when it kicks in: manufacturing plant, flowering plant)

# Complex Features with NB?

- Example: Washington County jail served 11,166 meals last month - a figure that translates to feeding some 120 people three times daily for 31 days.
- So we have a decision to make based on a set of cues:
  - context:jail, context:county, context:feeding, context:meals, …
  - subcat:NP, direct-object-head:meals
- Not clear how build a generative derivation for these:
  - Choose topic, then decide on having a transitive usage, then pick “meals” to be the object’s head, then generate other words?
  - Hard to make this work (though maybe possible)
  - No real reason to try

# A Discriminative Approach

- View WSD as a discrimination task, directly estimate:

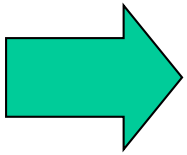
  P(sense | context:jail, context:county,
  context:feeding, context:meals, …
  subcat:NP, direct-object-head:meals, ….)

- Have to estimate multinomial (over senses) where there are a huge number of things to condition on

- Many feature-based classification techniques out there
  - Log-linear models extremely popular in 2nd gen NLP community!

# Feature Representations

Washington County jail served 11,166 meals last month - a figure that translates to feeding some 120 people three times daily for 31 days.



context:jail = 1
context:county = 1
context:feeding = 1
context:game = 0
…
local-context:jail = 1
local-context:meals = 1
…
subcat:NP = 1
subcat:PP = 0
…
object-head:meals = 1
object-head:ball = 0

- Features are indicator functions which count the occurrences of certain patterns in the input
- *We will have different feature values for every pair of input x and class y*

# Features

- In NLP uses, usually a feature specifies
  1. an indicator function – a yes/no boolean matching function – of properties of the input and
  2. a particular class

$$\phi_i(x,y) \equiv [\Phi(x) \wedge y = y_j] \qquad \text{[Value is 0 or 1]}$$

  - Each feature picks out a data subset and suggests a label for it

# Example of Features

- context:jail & served:functional
- context:jail & served:dish
- …

- subcat:NP & served:functional
- subcat:NP & served:dish
- …

# Feature-Based Linear Classifiers

- Linear classifiers at classification time:
  - Linear function from feature sets $\{\phi_i\}$ to classes $\{y\}$.
  - Assign a weight $w_i$ to each feature $\phi_i$.
  - We consider each class for an observed datum $x$
  - For a pair $(x,y)$, features vote with their weights:
    - vote(y) = $\Sigma w_i \phi_i(x,y)$
    - Choose the class $y$ which maximizes $\Sigma w_i \phi_i(x,y)$
  - We need probabilistic semantics to this method.
    - Log linear classifiers

# Exponential Models (log-linear, maxent, Logistic, Gibbs)

- Model: use the scores as probabilities:

$$p(y|x;w) = \frac{\exp\left(w \cdot \phi(x,y)\right)}{\sum_{y'} \exp\left(w \cdot \phi(x,y')\right)}$$

Make positive

Normalize

- Learning: maximize the (log) conditional likelihood of training data $\{(x_i, y_i)\}_{i=1}^n$

$$L(w) = \sum_{i=1}^{n} \log p(y_i|x_i;w) \qquad w^* = \arg\max_w L(w)$$

  - Prediction: output $\text{argmax}_y$ p(y|x;w)

# Feature-Based Linear Classifiers

- Exponential (log-linear, maxent, logistic, Gibbs) models:
  - Given this model form, we will choose parameters $\{w_i\}$ that *maximize the conditional likelihood* of the data according to this model.
  - We construct not only classifications, but probability distributions over classifications.
    - There are other (good!) ways of discriminating classes – SVMs, boosting, even perceptrons – but these methods are not as trivial to interpret as distributions over classes.

# Derivative of Log-linear Model

$$p(y|x;w) = \frac{\exp\left(w \cdot \phi(x,y)\right)}{\sum_{y'} \exp\left(w \cdot \phi(x,y')\right)}$$

- Unfortunately, $\text{argmax}_w$ L(w) doesn't have a close formed solution
- We will have to differentiate and use gradient ascent

$$L(w) = \sum_{i=1}^{n} \log p(y_i|x_i;w)$$

$$L(w) = \sum_{i=1}^{n} \left( w \cdot \varphi(x_i,y_i) - \log \sum_{y} \exp(w \cdot \varphi(x_i,y)) \right)$$

$$\frac{\partial L(w)}{\partial w_{jk}} = \sum_{i=1}^{n} \left( \varphi_{jk}(x_i,y_i) - \mathrm{p(k}|x_i;\mathrm{w})\varphi_{jk}(x_i,\mathrm{k}) \right)$$

Total count of feature j in correct candidates

Expected count of feature j in predicted candidates

# Proof (Conditional Likelihood Derivative)

- Recall

$$p(y|x;w) = \frac{\exp\left(w \cdot \phi(x,y)\right)}{\sum_{y'} \exp\left(w \cdot \phi(x,y')\right)}$$

$$P(Y \mid X, w) = \prod_{(x,y)\in D} p(y \mid x, w)$$

- We can separate this into two components:

$$\log P(Y|X,w) = \sum_{i=1}^{n} (w \cdot \varphi(x_i,y_i)) - \sum_{i=1}^{n} \left( \log \sum_{y} \exp(w \cdot \varphi(x_i,y)) \right)$$

- The derivative is the difference between the derivatives of each component

$$\log P(Y \mid X, w) = N(w) - D(w)$$

# Proof: Numerator

$$\frac{\partial N(w)}{\partial w_{jk}} = \frac{\partial \sum_{i=1}^{n}\left(\sum_{l}\left(w_{ly_i}\varphi_{ly_i}(x_i,y_i)\right)\right)}{\partial w_{jk}}$$

$$= \sum_{i=1}^{n} \frac{\partial\left(\sum_{l}\left(w_{ly_i}\varphi_{ly_i}(x_i,y_i)\right)\right)}{\partial w_{jk}}$$

$$= \sum_{i=1}^{n} \varphi_{jk}\,(x_i,y_i)$$

Derivative of the numerator is:

the empirical count of feature j with class k

Note: $\varphi_{jk}\,(x_i,y_i)$=0 if y≠ $k$

# Proof: Denominator

$$\frac{\partial D(w)}{\partial w_{jk}} = \frac{\partial \sum_{i=1}^{n} \log \sum_y \exp\left(\sum_l \left(w_{ly} \varphi_{ly}\ (x_i, y)\right)\right)}{\partial w_{jk}}$$

$$= \sum_{i=1}^{n} \frac{1}{\sum_{y'} \exp\left(\sum_l \left(w_{ly'} \varphi_{ly'}\ (x_i, y')\right)\right)} \frac{\partial \sum_y \exp\left(\sum_l \left(w_{ly} \varphi_{ly}\ (x_i, y)\right)\right)}{\partial w_{jk}}$$

$$= \sum_{i=1}^{n} \frac{1}{\sum_{y'} \exp\left(\sum_l \left(w_{ly'} \varphi_{ly'}\ (x_i, y')\right)\right)} \sum_y \frac{\exp\left(\sum_l \left(w_{ly} \varphi_{ly}\ (x_i, y)\right)\right)}{1} \frac{\partial \sum_l \left(w_{ly} \varphi_{ly} (x_i, y)\right)}{\partial w_{jk}}$$

$$= \sum_{i=1}^{n} \sum_y \frac{\exp\left(\sum_l \left(w_{ly} \varphi_{ly}\ (x_i, y)\right)\right)}{\sum_{y'} \exp\left(\sum_l \left(w_{ly'} \varphi_{ly'}\ (x_i, y')\right)\right)} \varphi_{jk}\ (x_i, y)$$

$$= \sum_{i=1}^{n} \sum_y P(y | x_i; w)\ \varphi_{jk}\ (x_i, y)$$

$$= \sum_{i=1}^{n} \mathrm{p}(\mathrm{k} | x_i; \mathrm{w}) \varphi_{jk}(x_i, \mathrm{k})$$

= expected count of
feature j predicted with class k

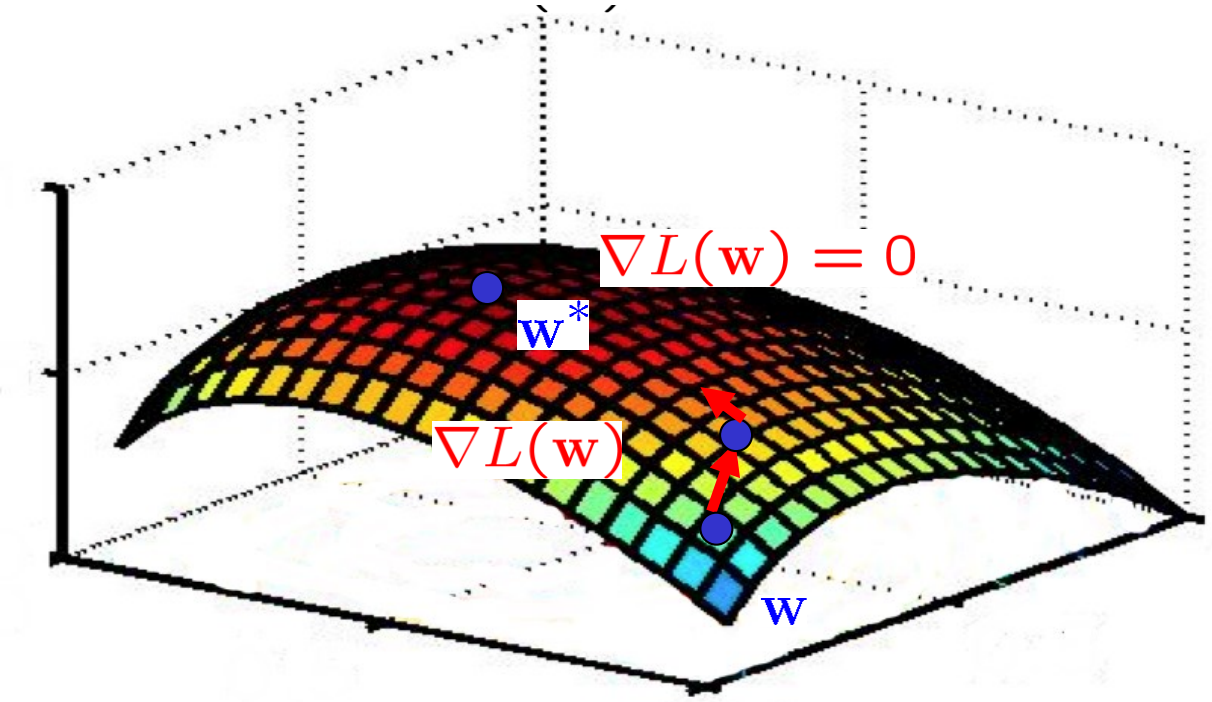
# Proof (concluded)

$$\frac{\partial P(Y|X;w)}{\partial w_{jk}} = actualcount(\varphi_{jk}) - \text{predictedcount}(\varphi_{jk})$$

- The optimum parameters are the ones for which each feature's predicted expectation equals its empirical expectation. The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if feature counts are from actual data).
- These models are also called maximum entropy models because we find the model has the maximum entropy while satisfying the constraints:

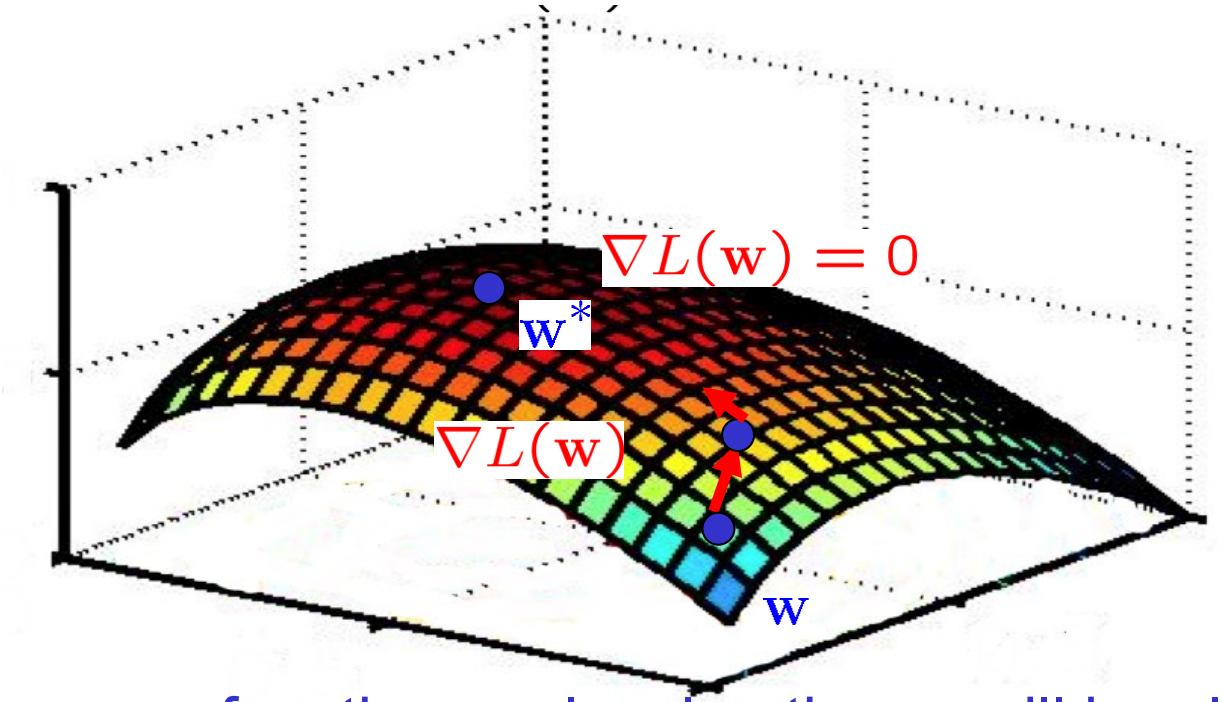
$$E_p(\phi_i) = E_{\tilde{p}}(\phi_i), \forall i$$

# Unconstrained Optimization




- Basic idea: move uphill from current guess
- Gradient ascent / descent follows the gradient incrementally
- At local optimum, derivative vector is zero
- Will converge if step sizes are small enough, but not efficient
- All we need is to be able to evaluate the function and its derivative

# Unconstrained Optimization




- For convex functions, a local optimum will be global
- Basic gradient ascent isn't very efficient, but there are simple enhancements which take into account previous gradients: conjugate gradient, L-BFGS
- There are special-purpose optimization techniques for maxent, like iterative scaling, but they aren't better

# What About Overfitting?

- For Naïve Bayes, we were worried about zero counts in MLE estimates
  - Can that happen here?

- Regularization (smoothing) for Log-linear models
  - Instead, we worry about large feature weights
  - Add a regularization term to the likelihood to push weights towards zero

$$L(w) = \sum_{i=1}^{n} \log p(y_i|x_i; w) - \frac{\lambda}{2}||w||^2$$

# Derivative for Regularized Maximum Entropy

- Unfortunately, $\text{argmax}_w$ L(w) still doesn't have a close formed solution
- We will have to differentiate and use gradient ascent

$$L(w) = \sum_{i=1}^{n} \left( w \cdot \phi(x_i, y_i) - \log \sum_y \exp(w \cdot \phi(x_i, y)) \right) - \frac{\lambda}{2} ||w||^2$$

$$\frac{\partial}{\partial w_j} L(w) = \sum_{i=1}^{n} \left( \phi_j(x_i, y_i) - \sum_y p(y|x_i; w) \phi_j(x_i, y) \right) - \lambda w_j$$

Total count of feature j in correct candidates

Expected count of feature j in predicted candidates

Big weights are bad

# L1 and L2 Regularization

## L2 Regularization for Log-linear models

- Instead, we worry about large feature weights
- Add a regularization term to the likelihood to push weights towards zero

$$L(w) = \sum_{i=1}^{n} \log p(y_i|x_i; w) - \frac{\lambda}{2}||w||^2$$

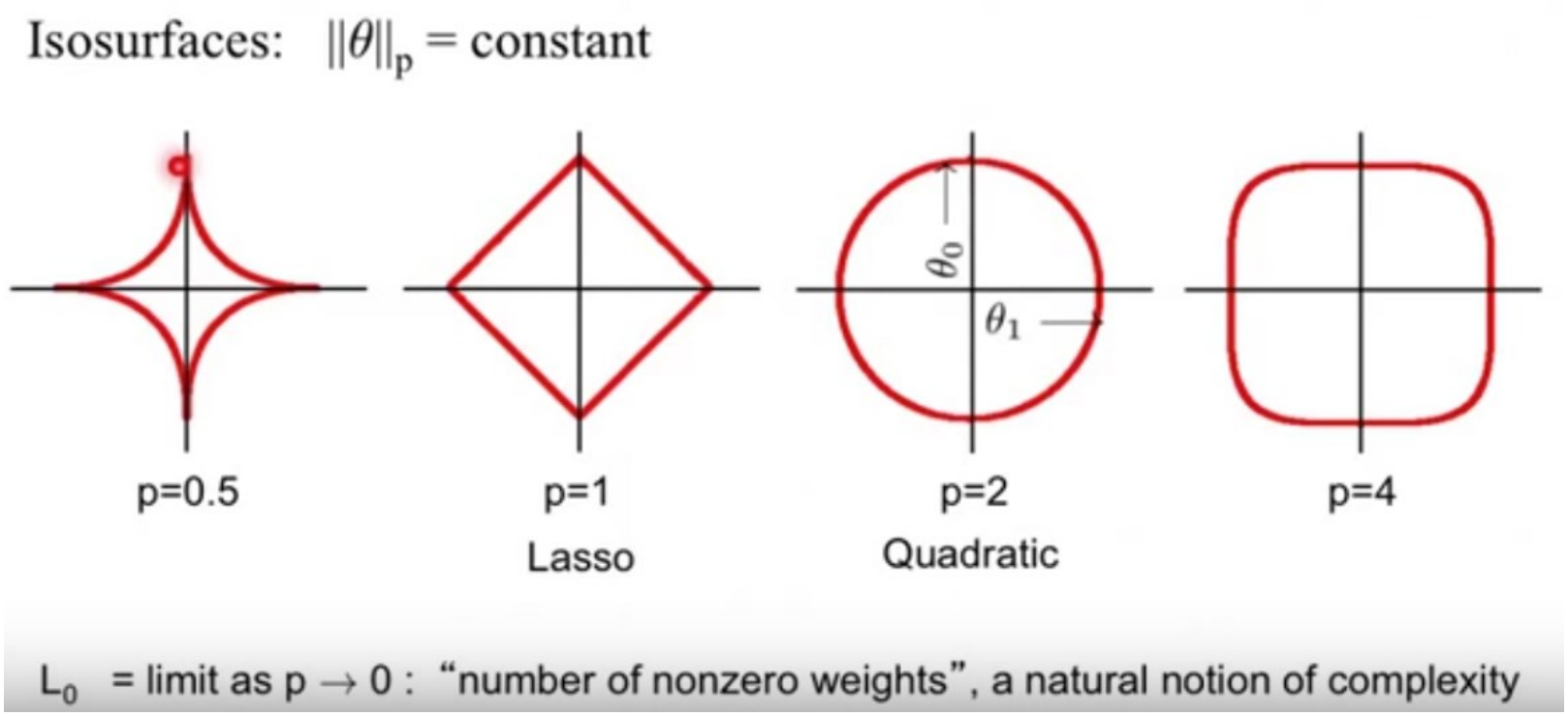
Regularization Constant

## L1 Regularization for Log-linear models

- Instead, we worry about number of active features
- Add a regularization term to the likelihood to push weights to zero

$$L(w) = \sum_{i=1}^{n} \log p(y_i|x_i; w) - \boldsymbol{\lambda}\,|w||$$

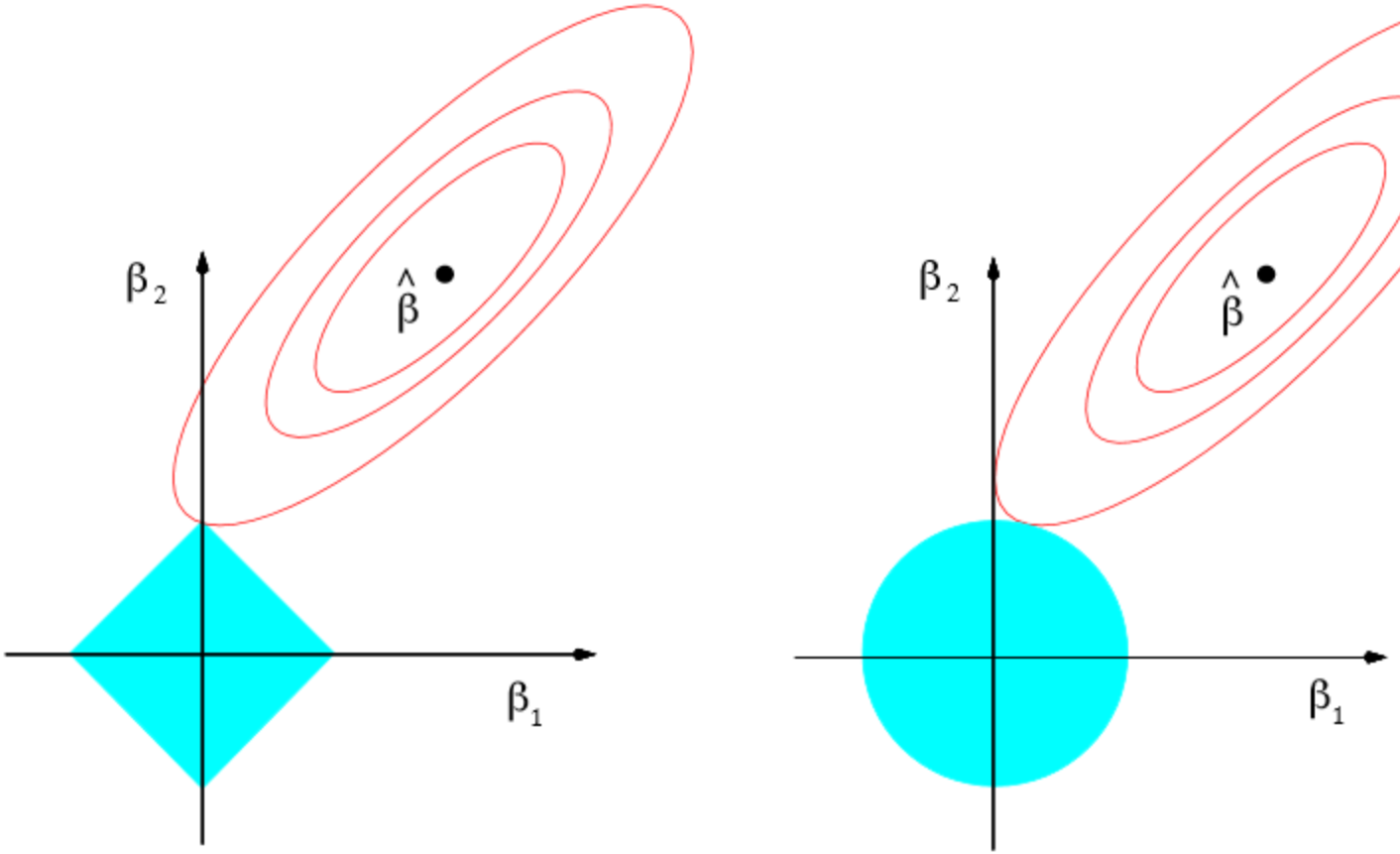
# $L_p$ Norms for Regularization

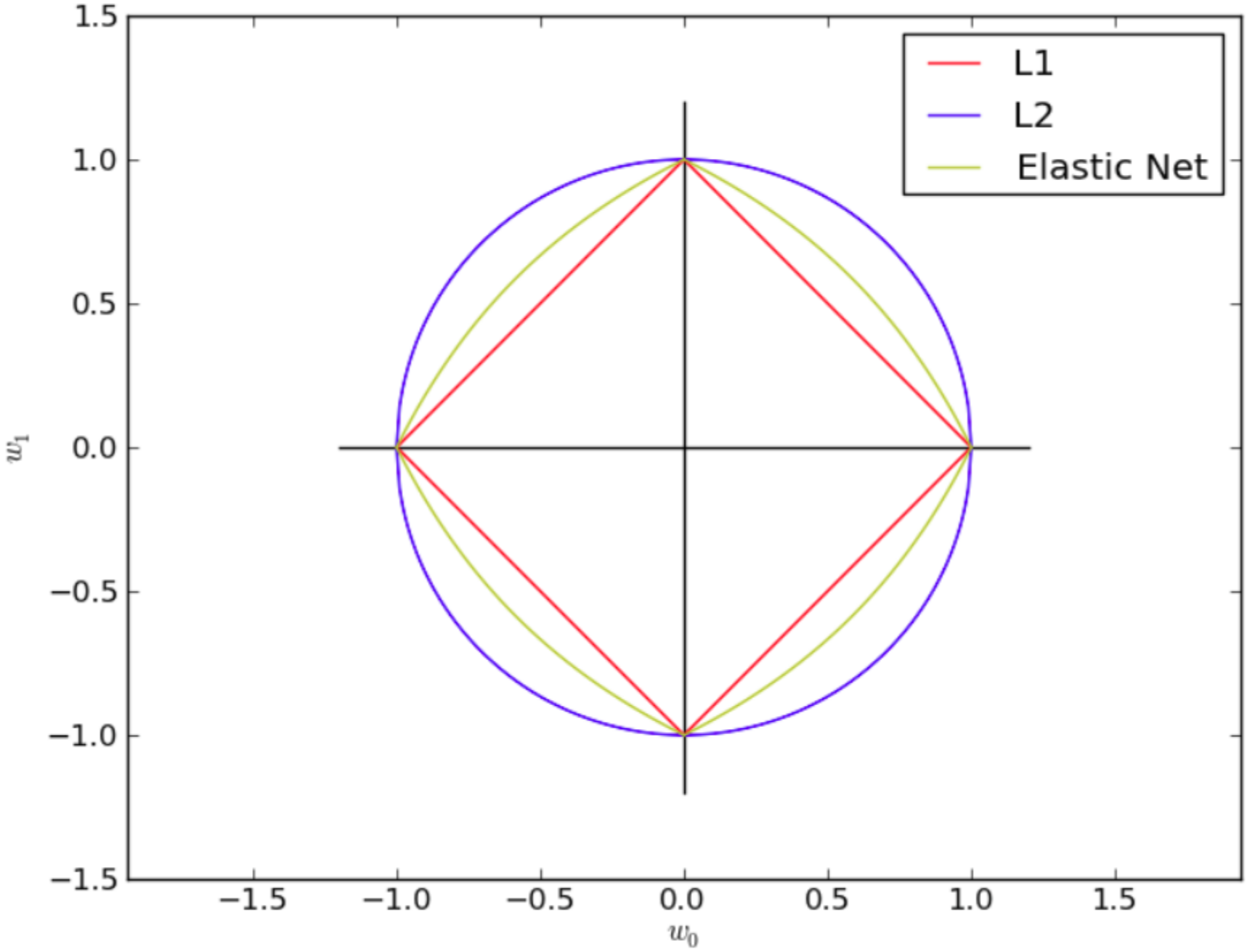



$L_0$ = limit as $p \to 0$ : "number of nonzero weights", a natural notion of complexity

# L1 vs L2

- Optimizing L1 harder
  - Discontinuous objective function
  - Subgradient descent versus gradient descent




L1
L2
Elastic Net
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
$w_1$
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
$w_0$

# How to pick weights?

- Goal: choose "best" vector w given training data
  - For now, we mean "best for classification"
- The ideal: the weights which have greatest test set accuracy / F1 / whatever
  - But, don't have the test set
  - Must compute weights from training set
- Maybe we want weights which give best training set accuracy?
  - May not (does not) generalize to test set
  - Easy to overfit
- Use devset

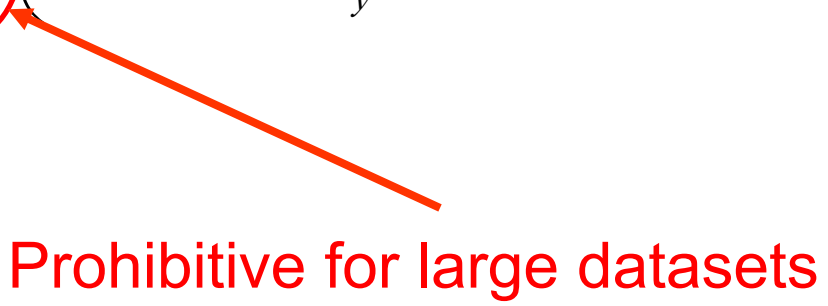
# Gradient Descent & Large Training Data

repeat

$$w^{(t+1)} \leftarrow w^{(t)} + \eta \frac{\partial L}{\partial w}$$

$$w_j^{(t+1)} \leftarrow w_j^{(t)} + \eta \left[ \frac{1}{N} \sum_{i=1}^{N} \left( \phi_j(x_i, y_i) - \sum_y p(y \mid x_i, w^{(t)}) \phi_j(x_i, y) \right) - \lambda w_j^{(t)} \right]$$

until convergence



# Stochastic Gradient Descent

repeat

$$w_j^{(t+1)} \leftarrow w_j^{(t)} + \eta\left[\frac{1}{N}\sum_{i=1}^{N}\left(\phi_j(x_i,y_i) - \sum_y p(y \mid x_i, w^{(t)})\phi_j(x_i,y)\right) - \lambda w_j^{(t)}\right]$$

until convergence

Use gradient at current point as approx. for avg gradient!

repeat

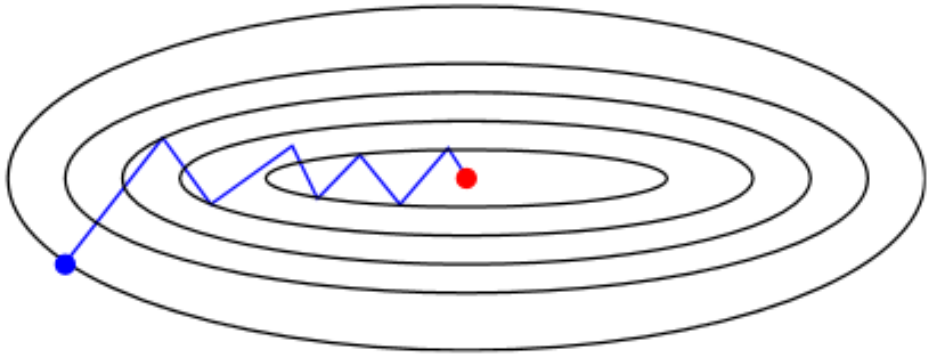
$$w_j^{(t+1)} \leftarrow w_j^{(t)} + \eta^{(t)}\left[\phi_j(x_i,y_i) - \sum_y p(y \mid x_i, w^{(t)})\phi_j(x_i,y) - \lambda w_j^{(t)}\right]$$

until convergence

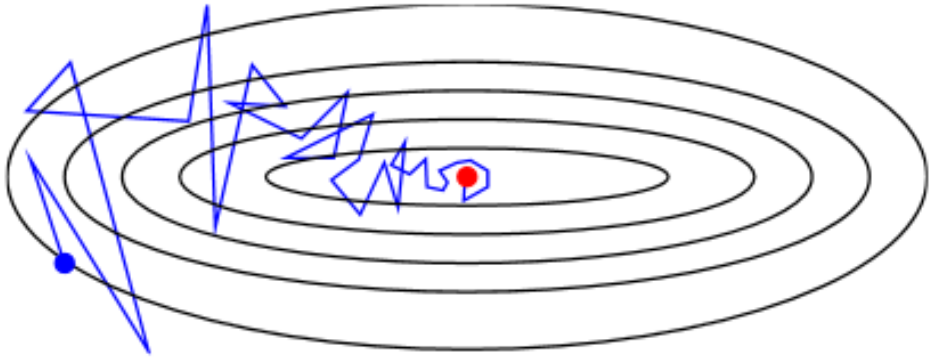
Reduce learning rate slowly (e.g., as η/t)

# SGD vs. GD

- Deterministic gradient method [Cauchy, 1847]:

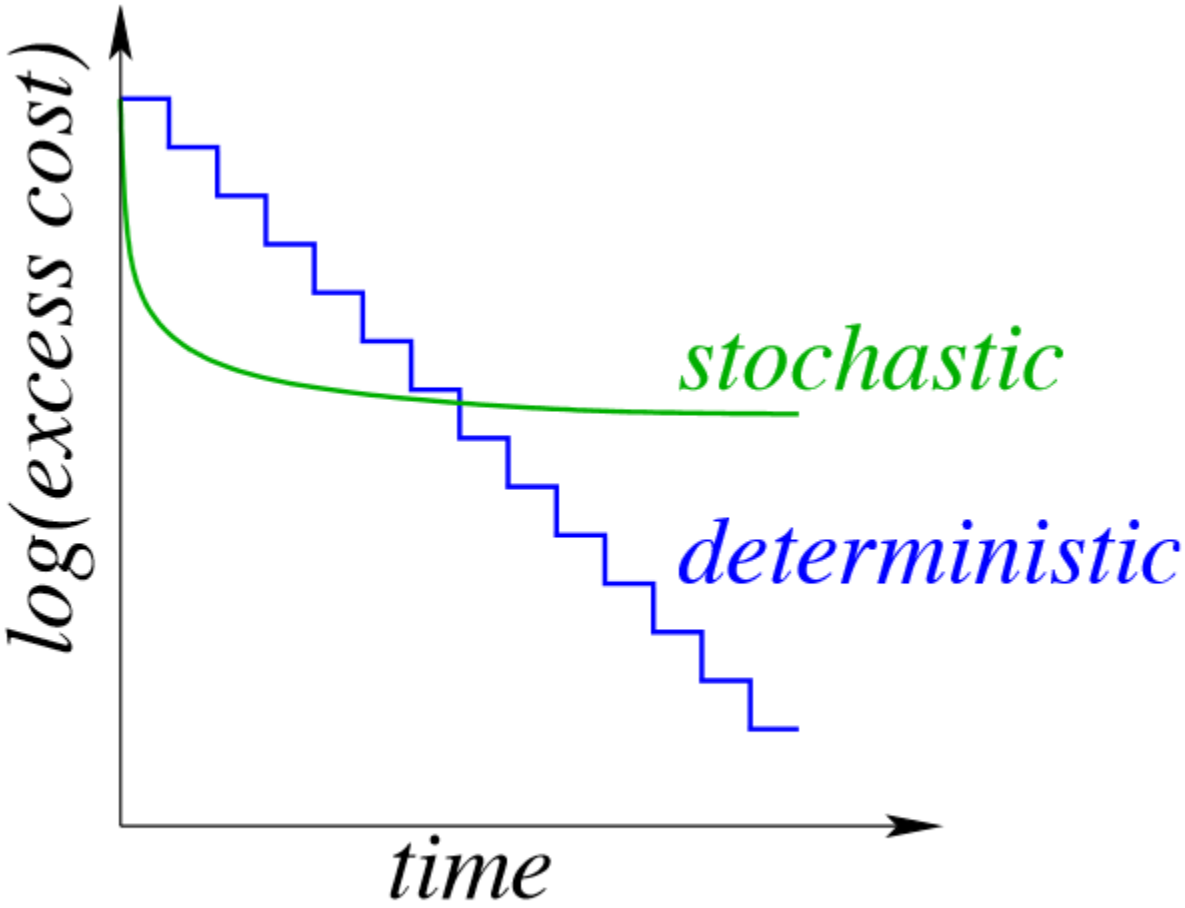


- Stochastic gradient method [Robbins & Monro, 1951]:



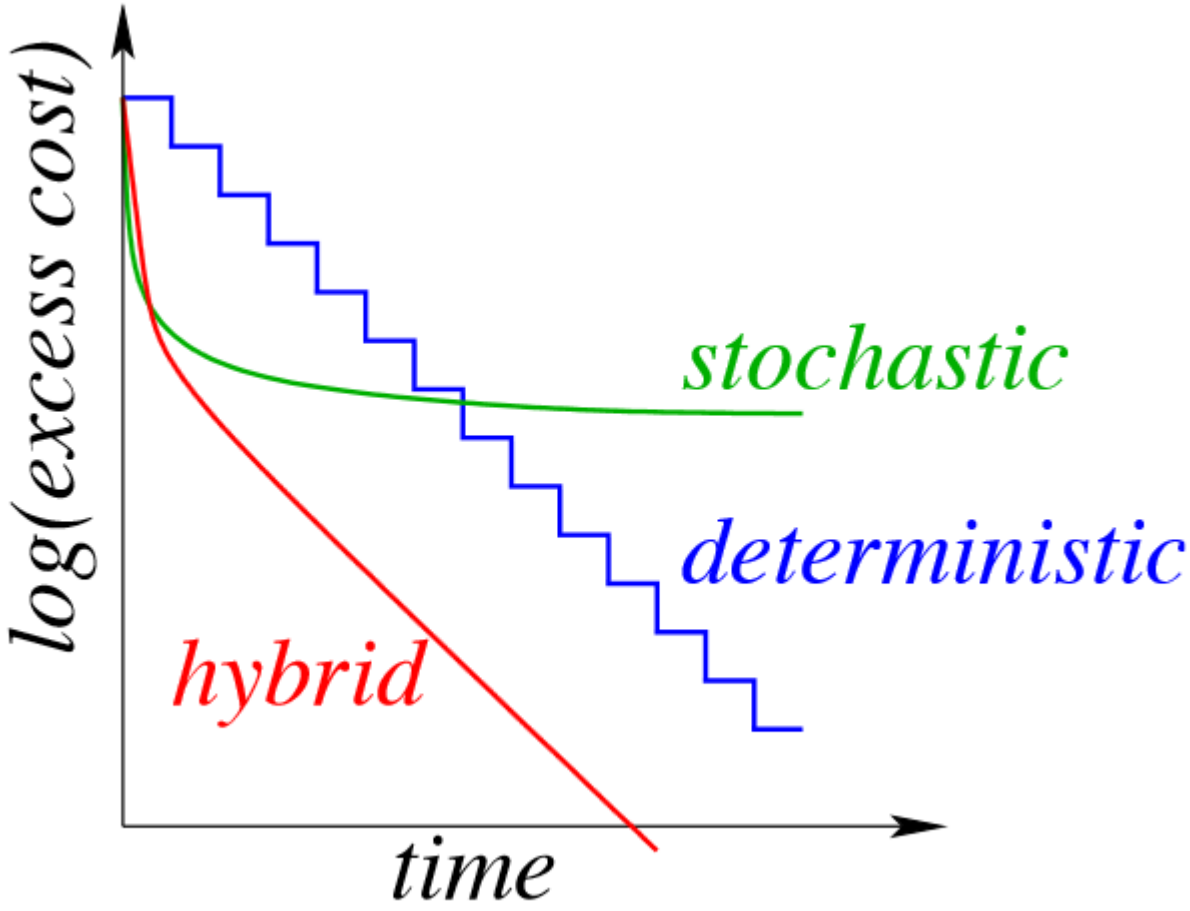
# Convergence rates

- GD: $O(1/t^2)$, SGD: $O(1/\sqrt{t})$




Stochastic will be superior for low-accuracy/time situations.

# Hybrid Approaches



# Hybrid: Batch

- Batch Gradient