

Logic in AI

Chapter 7

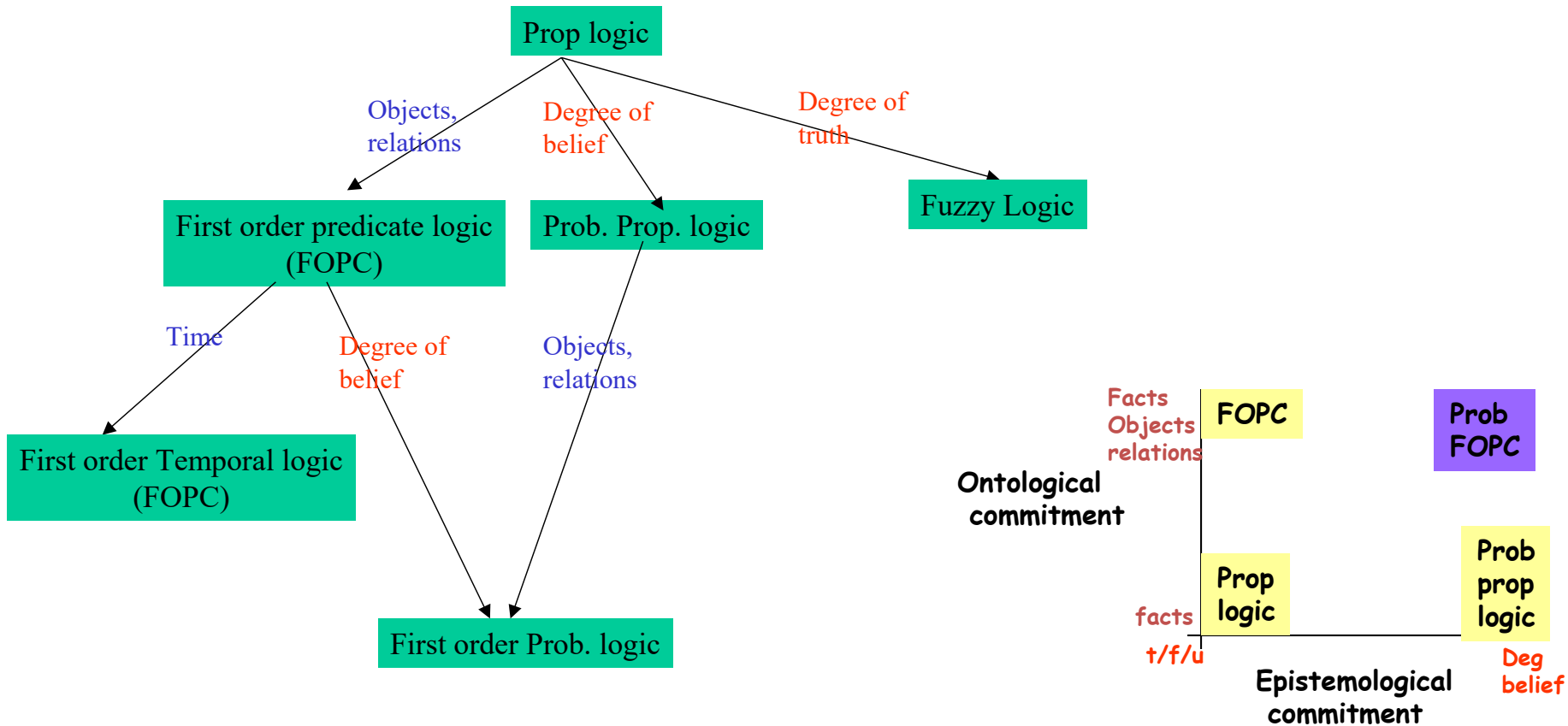
Mausam

(Based on slides of Dan Weld, Stuart Russell, Subbarao Kambhampati, Henry Kautz, Dieter Fox, and other UW AI Faculty)

Knowledge Representation

- *represent knowledge about the world in a manner that facilitates inferencing (i.e. drawing conclusions) from knowledge.*
- Example: Arithmetic logic
 - $x \geq 5$
- In AI: typically based on
 - Logic
 - Probability
 - Logic and Probability

Common KR Languages



KR Languages

- Propositional Logic
- Predicate Calculus
- Frame Systems
- Rules with Certainty Factors
- Bayesian Belief Networks
- Influence Diagrams
- Ontologies
- Semantic Networks
- Concept Description Languages
- Non-monotonic Logic

Basic Idea of Logic

- By starting with true assumptions, you can deduce true conclusions.

Truth

- Francis Bacon (1561-1626)

No pleasure is comparable to the standing upon the vantage-ground of truth.

- Thomas Henry Huxley (1825-1895)

Irrationally held truths may be more harmful than reasoned errors.

- John Keats (1795-1821)

Beauty is truth, truth beauty; that is all ye know on earth, and all ye need to know.

- Blaise Pascal (1623-1662)

We know the truth, not only by the reason, but also by the heart.

- François Rabelais (c. 1490-1553)

Speak the truth and shame the Devil.

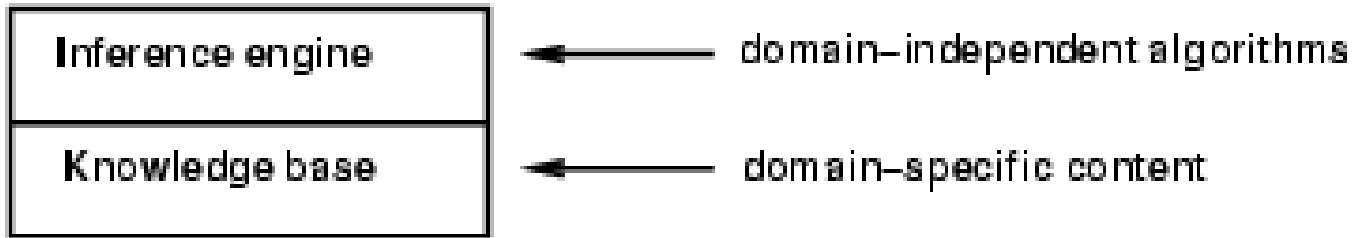
- Daniel Webster (1782-1852)

There is nothing so powerful as truth, and often nothing so strange.

Components of KR

- Syntax: defines the sentences in the language
- Semantics: defines the “meaning” to sentences
- Inference Procedure
 - Algorithm
 - Sound?
 - Complete?
 - Complexity
- Knowledge Base

Knowledge bases



- Knowledge base = set of **sentences** in a **formal** language
- **Declarative** approach to building an agent (or other system):
 - `Tell` it what it needs to know
- Then it can `Ask` itself what to do - answers should follow from the KB
- Agents can be viewed at the **knowledge level**
i.e., what they know, regardless of how implemented
- Or at the **implementation level**
i.e., data structures in KB and algorithms that manipulate them

Propositional Logic

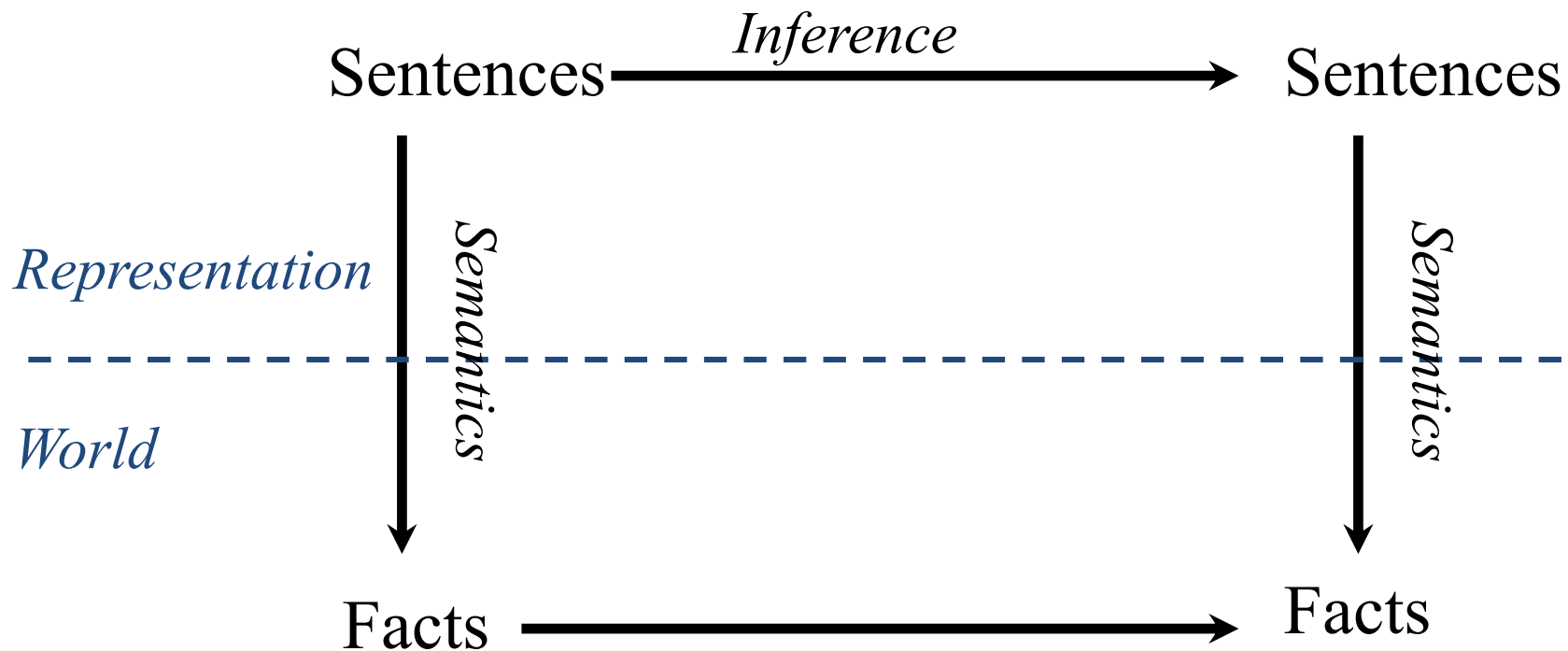
- Syntax
 - Atomic sentences: P, Q, \dots
 - Connectives: $\wedge, \vee, \neg, \rightarrow$
- Semantics
 - Truth Tables
- Inference
 - Modus Ponens
 - Resolution
 - DPLL
 - GSAT

Propositional Logic: Syntax

- Atoms
 - P, Q, R, \dots
- Literals
 - $P, \neg P$
- Sentences
 - Any literal is a sentence
 - If S is a sentence
 - Then $(S \wedge S)$ is a sentence
 - Then $(S \vee S)$ is a sentence
- Conveniences
 - $P \rightarrow Q$ same as $\neg P \vee Q$

Semantics

- **Syntax**: which arrangements of symbols are *legal*
 - (Def “sentences”)
- **Semantics**: what the symbols *mean* in the world
 - (Mapping between symbols and worlds)



Propositional Logic: SEMANTICS

- “Interpretation” (or “possible world”)
 - Assignment to each variable either T or F
 - Assignment of T or F to each connective via defns

		Q	
		T	F
P	T	T	F
	F	F	F

$P \wedge Q$

		Q	
		T	F
P	T	T	T
	F	F	F

$P \vee Q$

Satisfiability, Validity, & Entailment

- S is **satisfiable** if it is true in *some* world
- S is **unsatisfiable** if it is false in *all* worlds
- S is **valid** if it is true in *all* worlds
- S1 **entails** S2 if *whenever* S1 is true S2 is also true

Examples

$$P \rightarrow Q$$

$$R \rightarrow \neg R$$

$$S \wedge (W \wedge \neg S)$$

$$T \vee \neg T$$

$$X \rightarrow X$$

Notation

\Rightarrow

\cup

\rightarrow

\vdash

\models



Implication (syntactic symbol)

Proves: $S1 \vdash_{ie} S2$ if 'ie' algorithm says 'S2' from S1

Entails: $S1 \models S2$ if wherever S1 is true S2 is also true

• **Sound**

$\vdash \rightarrow \models$

• **Complete**

$\models \rightarrow \vdash$

• *(all truth & nothing but the truth)*

Reasoning Tasks

- **Model finding**

KB = background knowledge

S = description of problem

Show $(KB \wedge S)$ is satisfiable

A kind of **constraint satisfaction**

- **Deduction**

S = question

Prove that $KB \models S$

Two approaches:

- **Rules to derive new formulas from old (inference)**
- **Show $(KB \wedge \neg S)$ is unsatisfiable**

Special Syntactic Forms

- General Form:

$$((q \wedge \neg r) \rightarrow s) \wedge \neg (s \wedge t)$$

- Conjunction Normal Form (CNF)

$$(\neg q \vee r \vee s) \wedge (\neg s \vee \neg t)$$

Set notation: $\{ (\neg q, r, s), (\neg s, \neg t) \}$

empty clause $() = \textit{false}$

- Binary clauses: 1 or 2 literals per clause

$$(\neg q \vee r) \quad (\neg s \vee \neg t)$$

- Horn clauses: 0 or 1 positive literal per clause

$$(\neg q \vee \neg r \vee s) \quad (\neg s \vee \neg t)$$

$$(q \wedge r) \rightarrow s \quad (s \wedge t) \rightarrow \textit{false}$$

Propositional Logic: Inference

A *mechanical* process for computing new sentences

1. Backward & Forward Chaining
2. Resolution (Proof by Contradiction)
3. SAT
 1. Davis Putnam
 2. WalkSat

Inference 1: Forward Chaining

Forward Chaining

Based on rule of *modus ponens*

If know P_1, \dots, P_n & know $(P_1 \wedge \dots \wedge P_n) \rightarrow Q$

Then can conclude Q

Backward Chaining: search

start from the query and go backwards

Analysis

- Sound?
- Complete?

Can you prove
 $\{ \} \models Q \vee \neg Q$

- If KB has only Horn clauses & query is a single literal
 - Forward Chaining is complete
 - Runs linear in the size of the KB

Example

$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

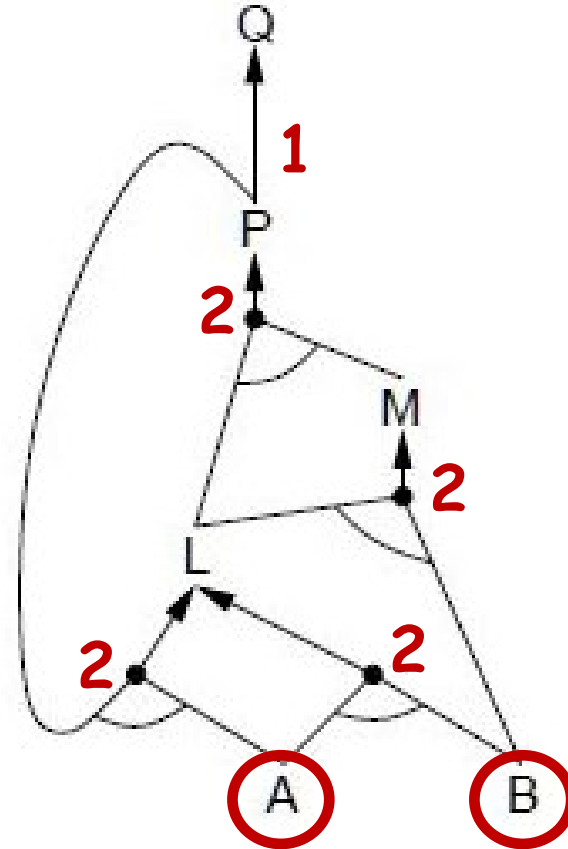
$$B \wedge L \Rightarrow M$$

$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

A

B



Example

$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

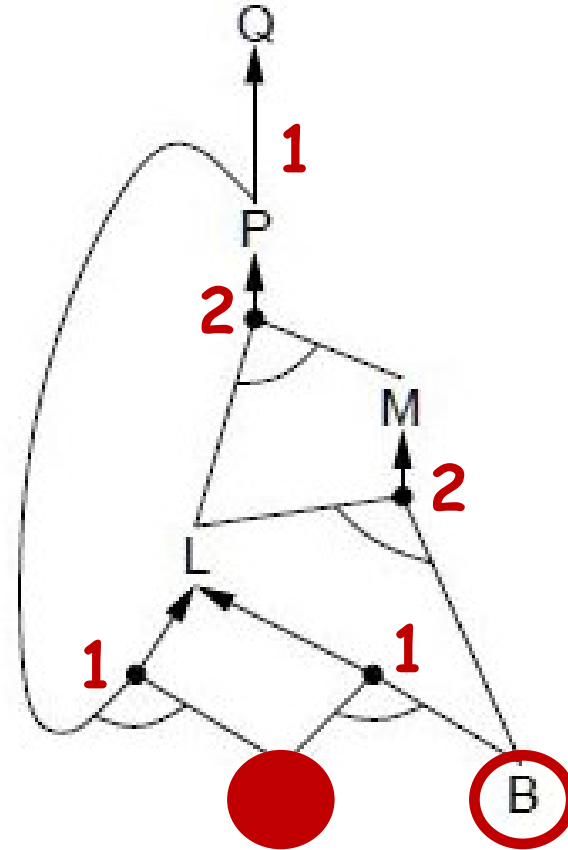
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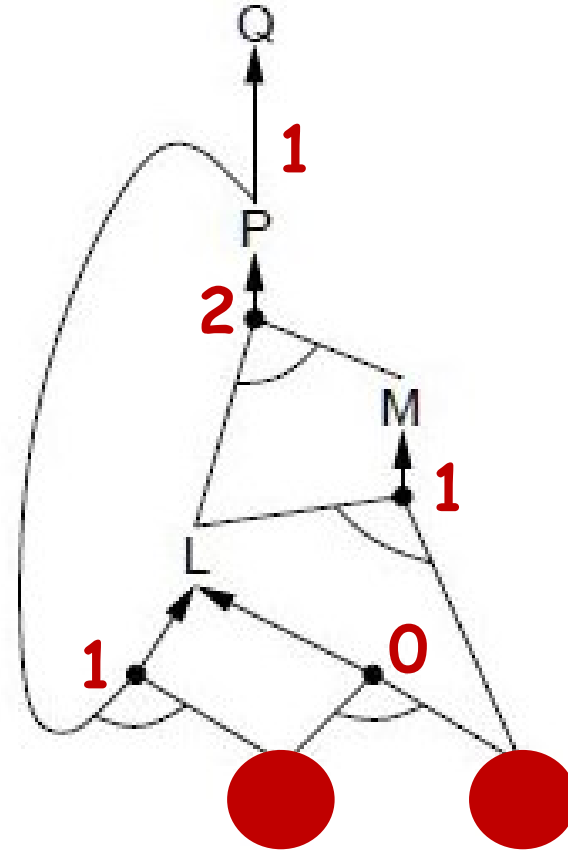
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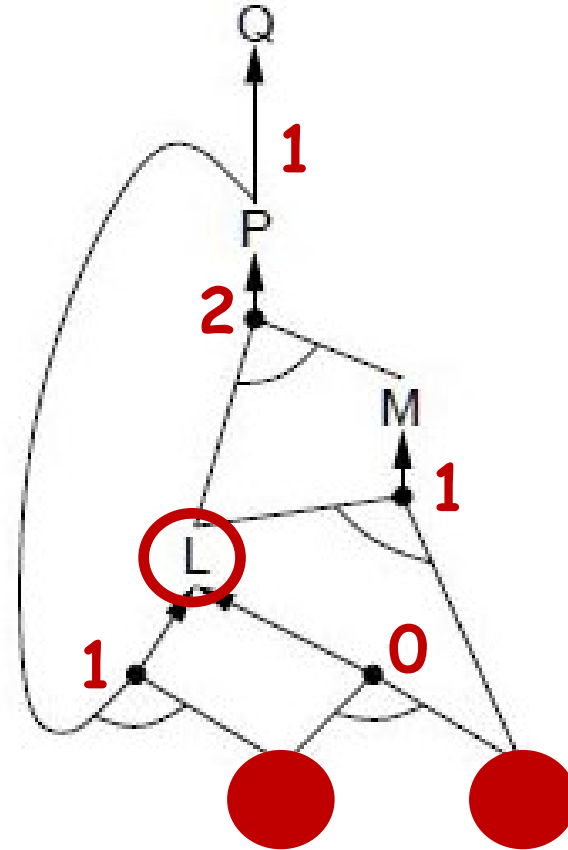
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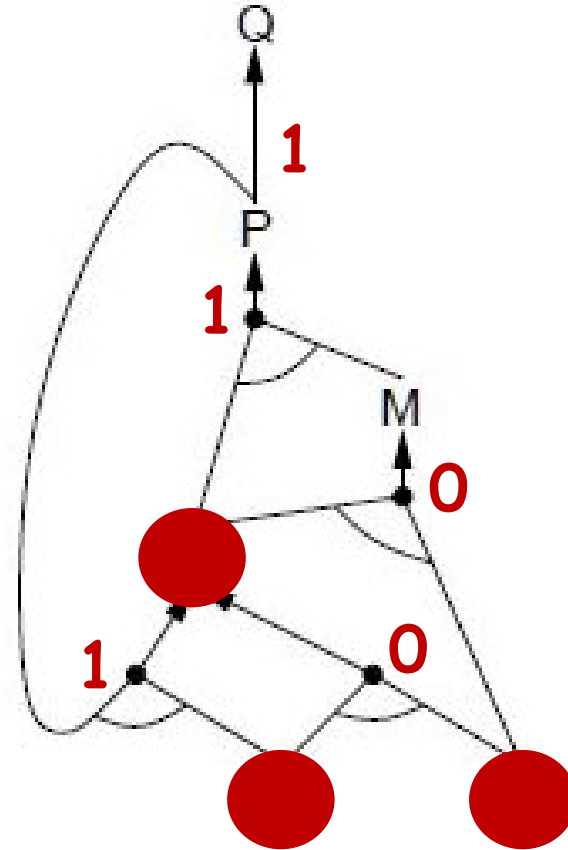
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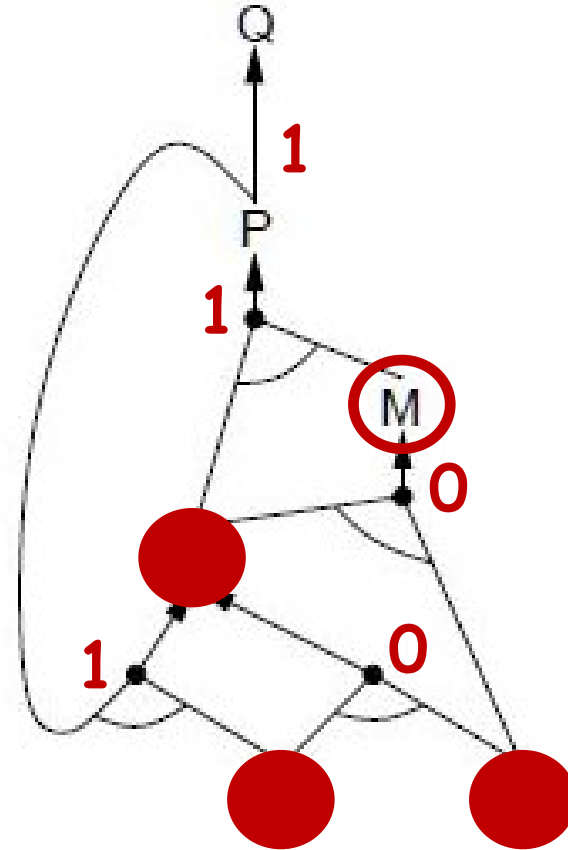
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Example

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$$L \wedge M \Rightarrow P$$

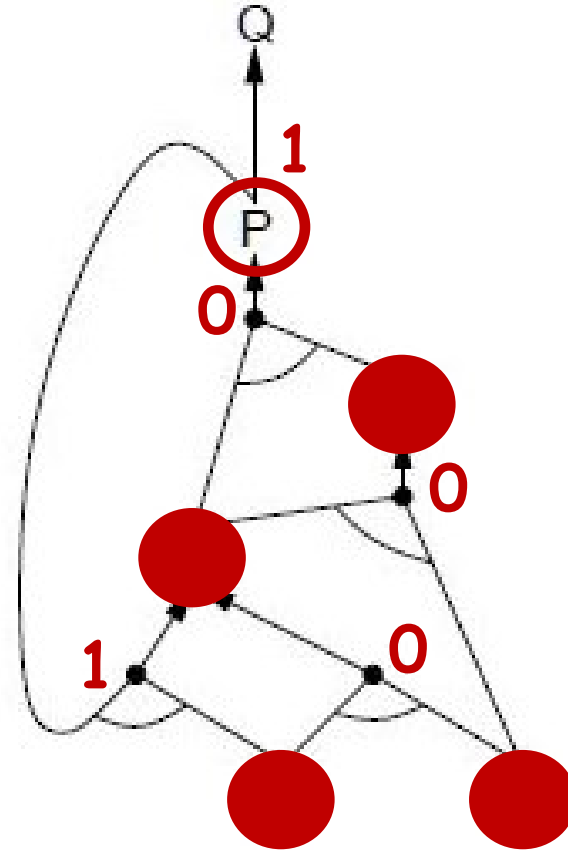
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$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

A

B



Example

$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

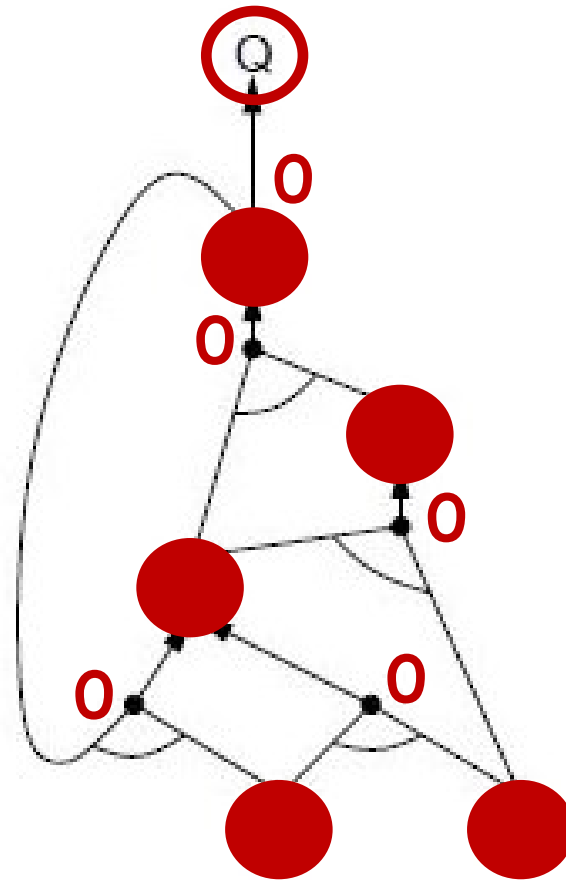
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Example

$$P \Rightarrow Q$$

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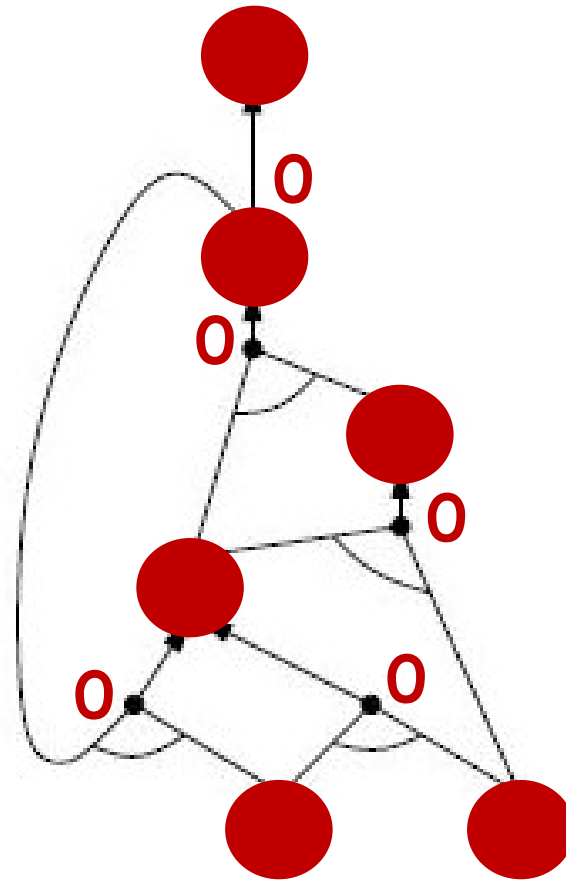
$$B \wedge L \Rightarrow M$$

$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

A

B



Propositional Logic: Inference

A *mechanical* process for computing new sentences

1. Backward & Forward Chaining
2. Resolution (Proof by Contradiction)
3. SAT
 1. Davis Putnam
 2. WalkSAT

Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move \neg inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law (\vee over \wedge) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Inference 2: Resolution

[Robinson 1965]

$$\{ (p \vee \alpha), (\neg p \vee \beta \vee \gamma) \} \vdash_{-R} (\alpha \vee \beta \vee \gamma)$$

Correctness

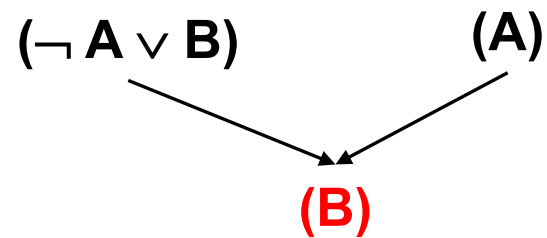
If $S1 \vdash_{-R} S2$ then $S1 \models S2$

Refutation Completeness:

If S is unsatisfiable then $S \vdash_{-R} ()$

Resolution subsumes Modus Ponens

$$A \rightarrow B, A \models B$$



If Will goes, Jane will go

$\sim W \vee J$

If doesn't go, Jane will still go

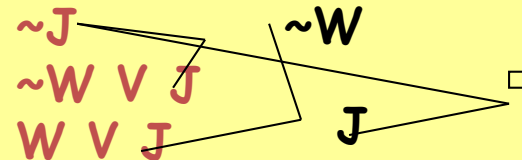
$W \vee J$

Will Jane go?

$\models J?$

$J \vee J = J$

Don't need to use other equivalences if we use resolution in *refutation* style

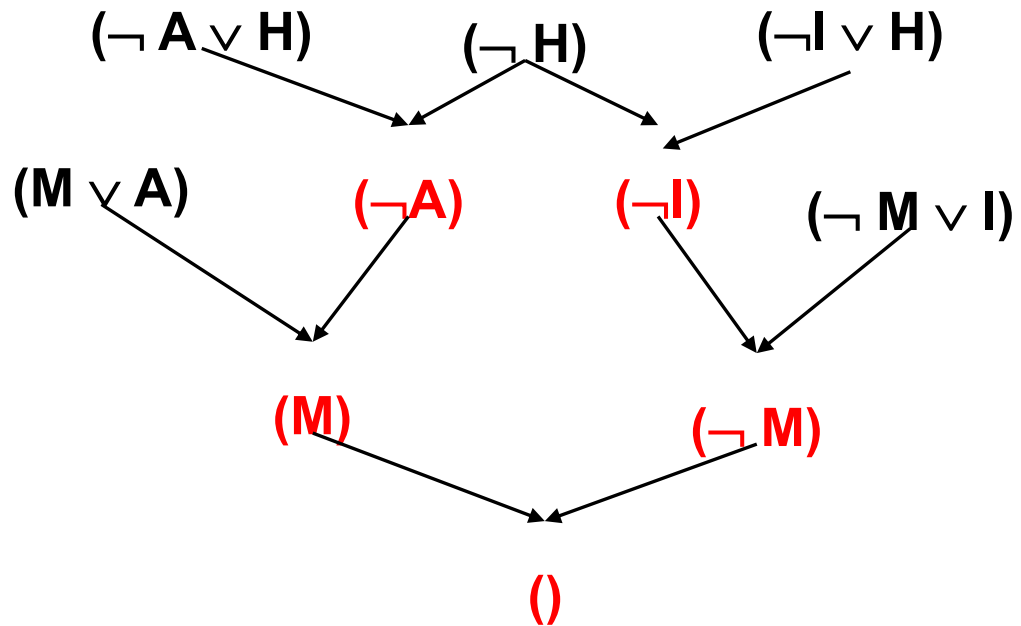


Resolution

If the unicorn is mythical, then it is immortal, but if it is not mythical, it is a mammal. If the unicorn is either immortal or a mammal, then it is horned.

Prove: the unicorn is horned.

M = mythical
I = immortal
A = mammal
H = horned



Search in Resolution

- Convert the database into clausal form D_c
- Negate the goal first, and *then* convert it into clausal form D_G
- Let $D = D_c + D_G$
- Loop
 - Select a pair of Clauses C1 and C2 from D
 - Different control strategies can be used to select C1 and C2 to reduce number of resolutions tries
 - Resolve C1 and C2 to get C12
 - If C12 is empty clause, QED!! Return Success (We proved the theorem;)
 - $D = D + C12$
- Out of loop but no empty clause. Return “Failure”
 - Finiteness is guaranteed if we make sure that:
 - we never resolve the same pair of clauses more than once;
 - we use factoring, which removes multiple copies of literals from a clause (e.g. QVPVP => QVP)

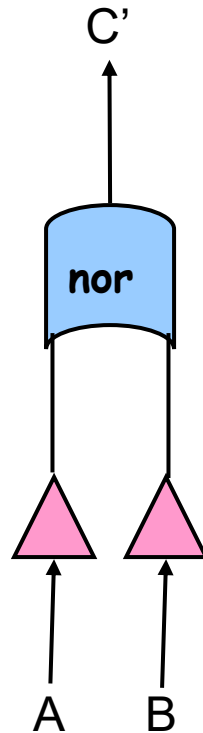
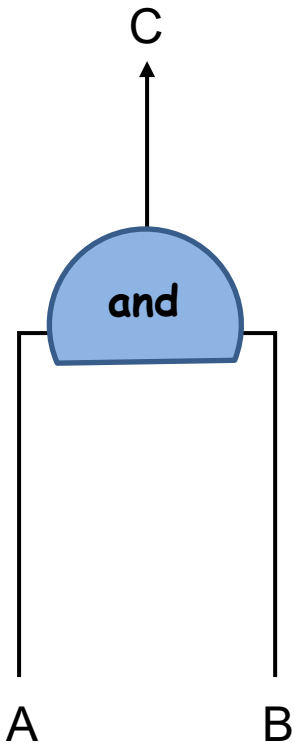
SAT: Model Finding

- Find assignments to variables that makes a formula true

Why study Satisfiability?

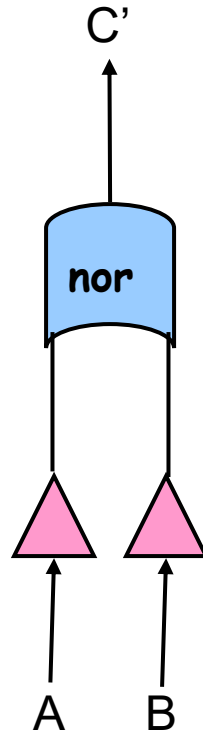
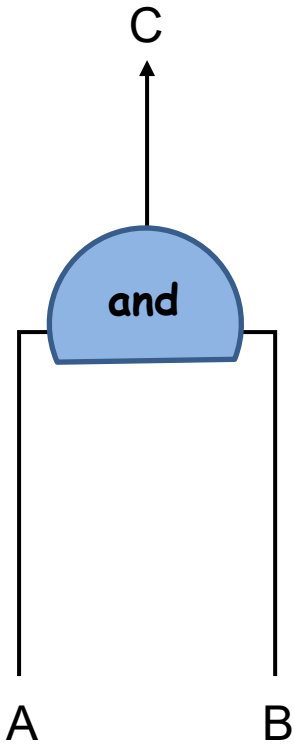
- Canonical NP complete problem.
 - several hard problems modeled as SAT
- Tonne of applications
- State-of-the-art solvers superfast

Testing Circuit Equivalence



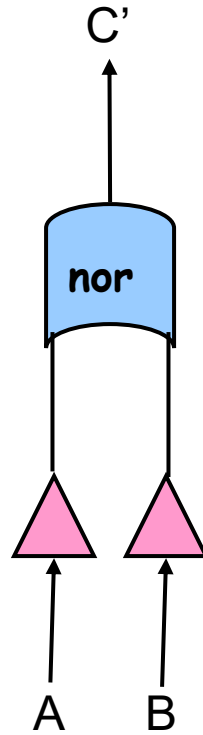
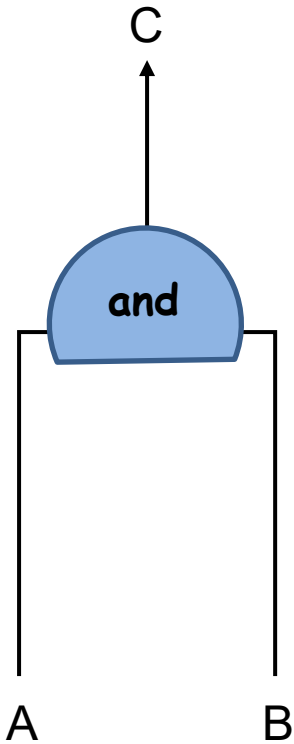
- Do two circuits compute the same function?
- Circuit optimization
- Is there input for which the two circuits compute different values?

Testing Circuit Equivalence



$$\begin{aligned} C &\equiv A \wedge B \\ C' &\equiv \neg(D \vee E) \\ D &\equiv \neg A \\ E &\equiv \neg B \end{aligned}$$

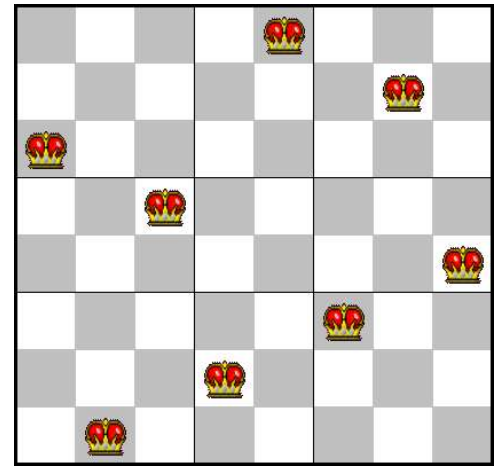
Testing Circuit Equivalence



$$\begin{aligned} C &\equiv A \wedge B \\ C' &\equiv \neg(D \vee E) \\ D &\equiv \neg A \\ E &\equiv \neg B \\ &\neg(C \equiv C') \end{aligned}$$

SAT Translation of N-Queens

- At least one queen each column:
($Q_{11} \vee Q_{12} \vee Q_{13} \vee \dots \vee Q_{18}$)
($Q_{21} \vee Q_{22} \vee Q_{23} \vee \dots \vee Q_{28}$)
...
- No attacks:
($\sim Q_{11} \vee \sim Q_{12}$)
($\sim Q_{11} \vee \sim Q_{22}$)
($\sim Q_{11} \vee \sim Q_{21}$)
...



Graph Coloring

- A new SAT Variable for var-val pair

$$X_{WA-r}, X_{WA-g}, X_{WA-b}, X_{NT-r} \dots$$

- Each var has at least 1 value

$$- X_{WA-r} \vee X_{WA-g} \vee X_{WA-b}$$

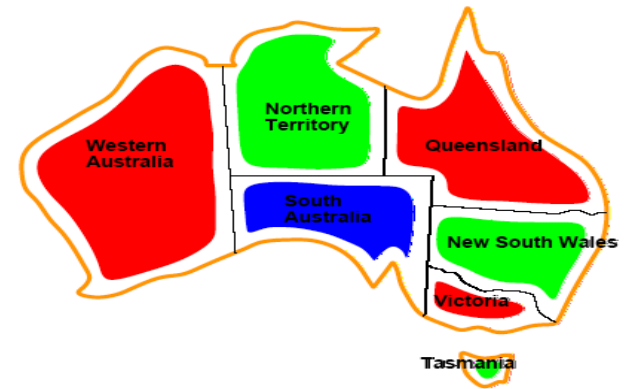
- No var has two values

$$- \sim X_{WA-r} \vee \sim X_{WA-g}$$

$$- \sim X_{WA-r} \vee \sim X_{WA-b}$$

- Constraints

$$- \sim X_{WA-r} \vee \sim X_{NT-r}$$



Application: Diagnosis

- Problem: diagnosis a malfunctioning device
 - Car
 - Computer system
 - Spacecraft
- where
 - Design of the device is known
 - We can observe the state of only certain parts of the device – much is hidden

Model-Based, Consistency-Based Diagnosis

- Idea: create a logical formula that describes how the device should work
 - Associated with each “breakable” component C is a proposition that states “C is okay”
 - Sub-formulas about component C are all conditioned on C being okay
- A diagnosis is a smallest of “not okay” assumptions that are consistent with what is actually observed

Consistency-Based Diagnosis

1. Make some **Observations** O .
2. Initialize the **Assumption Set** A to assert that all components are working properly.
3. Check if the KB , A , O together are **inconsistent** (can deduce *false*).
4. If so, delete propositions from A until **consistency is restored** (cannot deduce *false*).
The deleted propositions are a diagnosis.

There may be many possible diagnoses

Example: Automobile Diagnosis

- *Observable Propositions:*
EngineRuns, GasInTank, ClockRuns
- *Assumable Propositions:*
FuelLineOK, BatteryOK, CablesOK, ClockOK
- *Hidden (non-Assumable) Propositions:*
GasInEngine, PowerToPlugs
- *Device Description F:*
 $(\text{GasInTank} \wedge \text{FuelLineOK}) \rightarrow \text{GasInEngine}$
 $(\text{GasInEngine} \wedge \text{PowerToPlugs}) \rightarrow \text{EngineRuns}$
 $(\text{BatteryOK} \wedge \text{CablesOK}) \rightarrow \text{PowerToPlugs}$
 $(\text{BatteryOK} \wedge \text{ClockOK}) \rightarrow \text{ClockRuns}$
- *Observations:*
 $\neg \text{EngineRuns}, \text{GasInTank}, \text{ClockRuns}$

Example

- Is $F \cup \text{Observations} \cup \text{Assumptions}$ consistent?
- $F \cup \{\neg\text{EngineRuns}, \text{GasInTank}, \text{ClockRuns}\}$
 $\cup \{\text{FuelLineOK}, \text{BatteryOK}, \text{CablesOK}, \text{ClockOK}\} \rightarrow \text{false}$
 - *Must restore consistency!*
- $F \cup \{\neg\text{EngineRuns}, \text{GasInTank}, \text{ClockRuns}\}$
 $\cup \{\text{BatteryOK}, \text{CablesOK}, \text{ClockOK}\} \rightarrow \text{false}$
 - $\neg \text{FuelLineOK}$ is a diagnosis
- $F \cup \{\neg\text{EngineRuns}, \text{GasInTank}, \text{ClockRuns}\}$
 $\cup \{\text{FuelLineOK}, \text{CablesOK}, \text{ClockOK}\} \rightarrow \text{false}$
 - $\neg \text{BatteryOK}$ is not a diagnosis

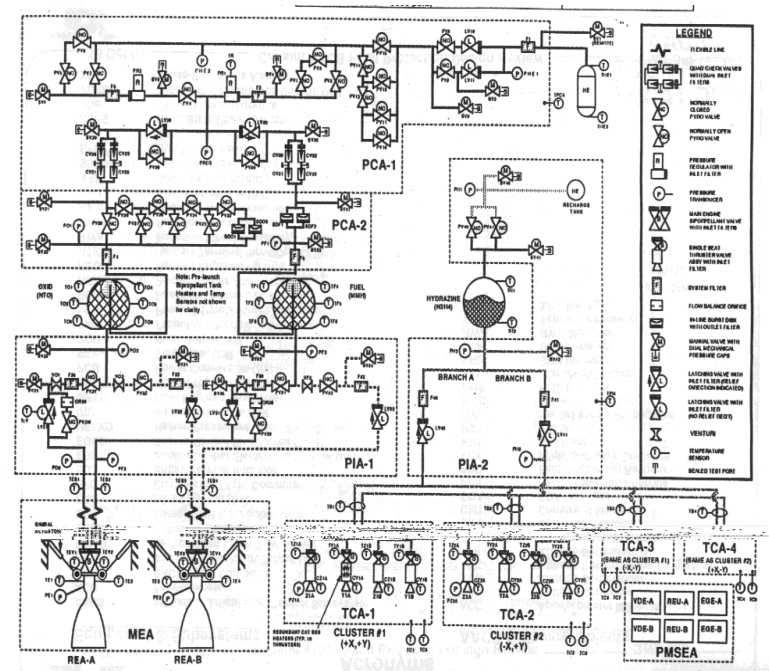
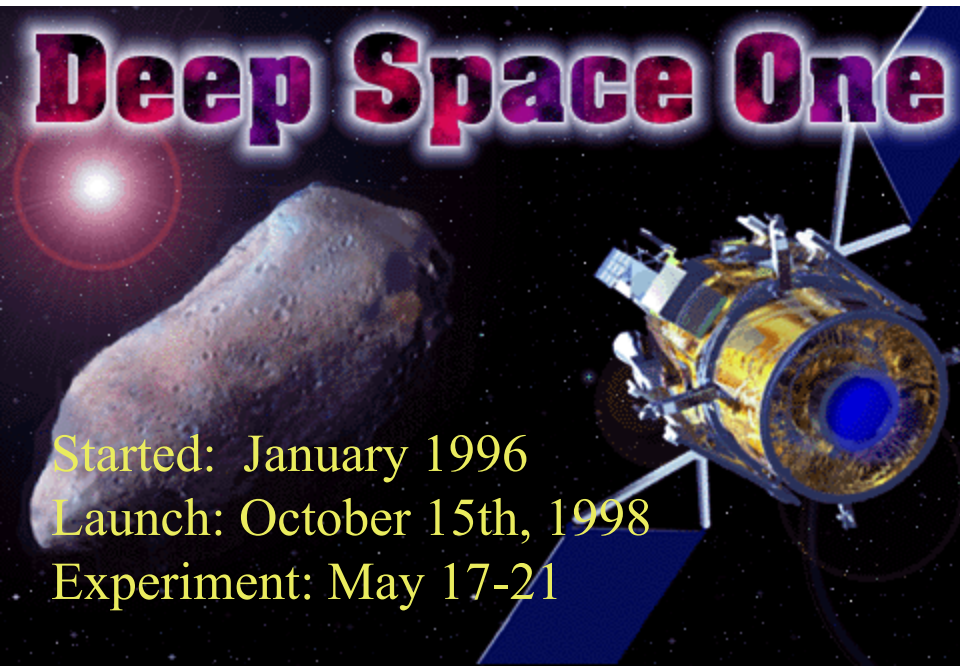
Complexity of Diagnosis

- If F is **Horn**, then each consistency test takes linear time – unit propagation is complete for Horn clauses.
- Complexity = ways to delete propositions from Assumption Set that are considered.
 - Single fault diagnosis – $O(n^2)$
 - Double fault diagnosis – $O(n^3)$
 - Triple fault diagnosis – $O(n^4)$

...

Deep Space One

- Autonomous diagnosis & repair “Remote Agent”
- Compiled systems schematic to 7,000 var SAT problem



Deep Space One

- a failed electronics unit
 - Remote Agent fixed by reactivating the unit.
- a failed sensor providing false information
 - Remote Agent recognized as unreliable and therefore correctly ignored.
- an attitude control thruster (a small engine for controlling the spacecraft's orientation) stuck in the "off" position
 - Remote Agent detected and compensated for by switching to a mode that did not rely on that thruster.

Inference 3: Model Enumeration

```
for (m in truth assignments) {  
    if (m makes  $\Phi$  true)  
        then return "Sat!"  
}  
return "Unsat!"
```

Inference 4: DPLL

(Enumeration of *Partial* Models)

[Davis, Putnam, Loveland & Logemann 1962]

Version 1

```
dp11_1(pa) {  
  if (pa makes F false) return false;  
  if (pa makes F true) return true;  
  choose P in F;  
  if (dp11_1(pa  $\cup$  {P=0})) return true;  
  return dp11_1(pa  $\cup$  {P=1});  
}
```

Returns true if F is satisfiable, false otherwise

DPLL Version 1

$$(a \vee b \vee c)$$

$$(a \vee \neg b)$$

$$(a \vee \neg c)$$

$$(\neg a \vee c)$$

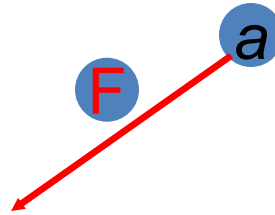
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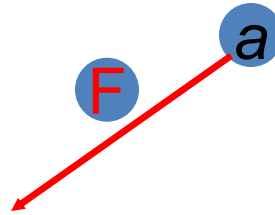
DPLL Version 1

$(F \vee b \vee c)$

$(F \vee \neg b)$

$(F \vee \neg c)$

$(T \vee c)$



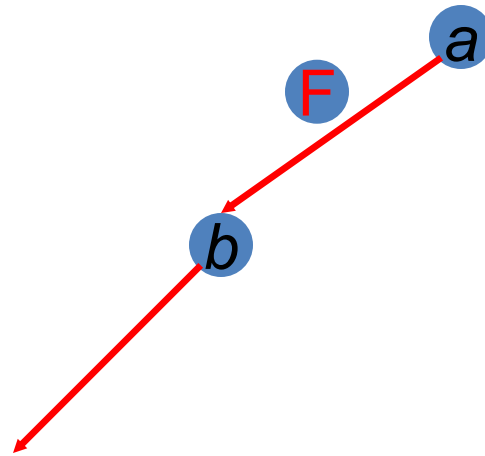
DPLL Version 1

$(F \vee F \vee c)$

$(F \vee T)$

$(F \vee \neg c)$

$(T \vee c)$



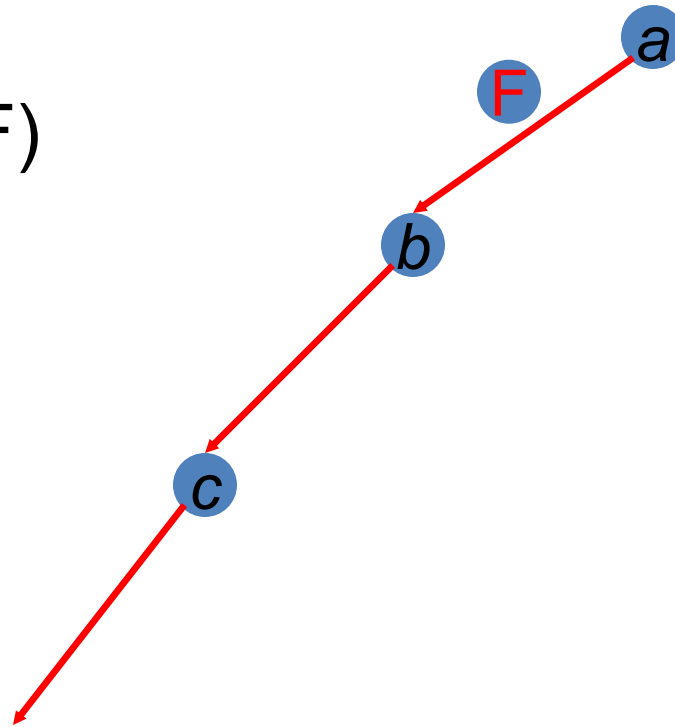
DPLL Version 1

$(F \vee F \vee F)$

$(F \vee T)$

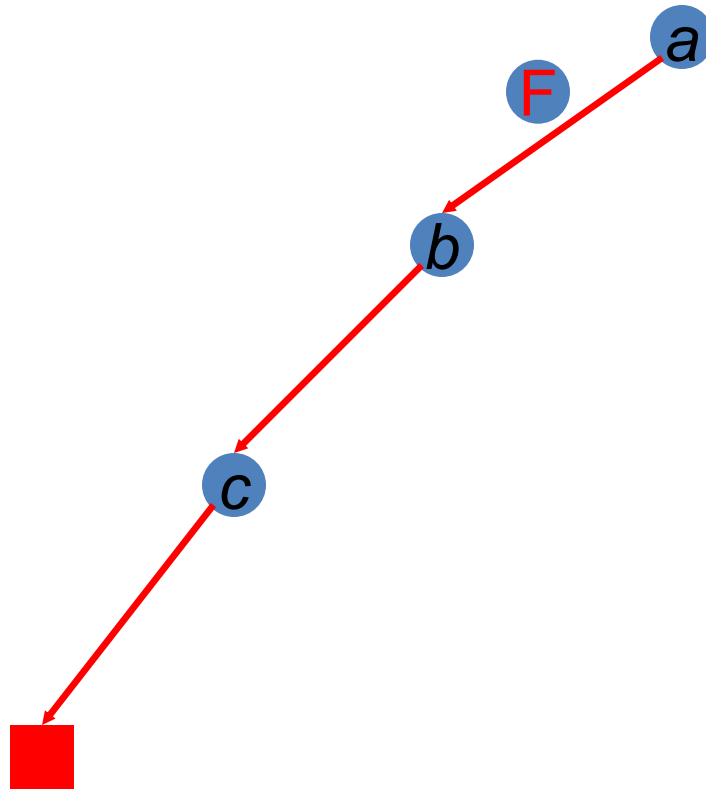
$(F \vee T)$

$(T \vee F)$



DPLL Version 1

F
T
T
T



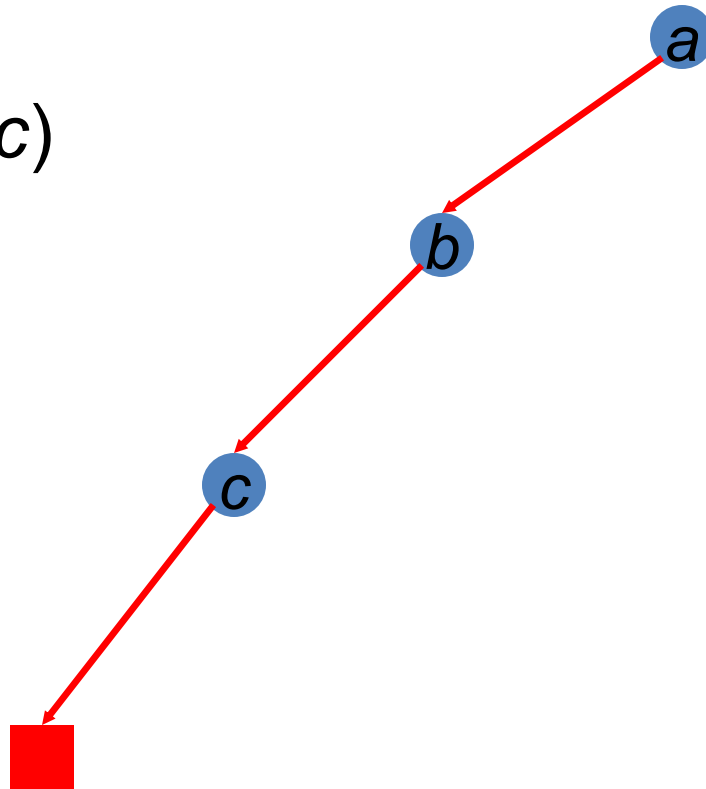
DPLL Version 1

$(a \vee b \vee c)$

$(a \vee \neg b)$

$(a \vee \neg c)$

$(\neg a \vee c)$



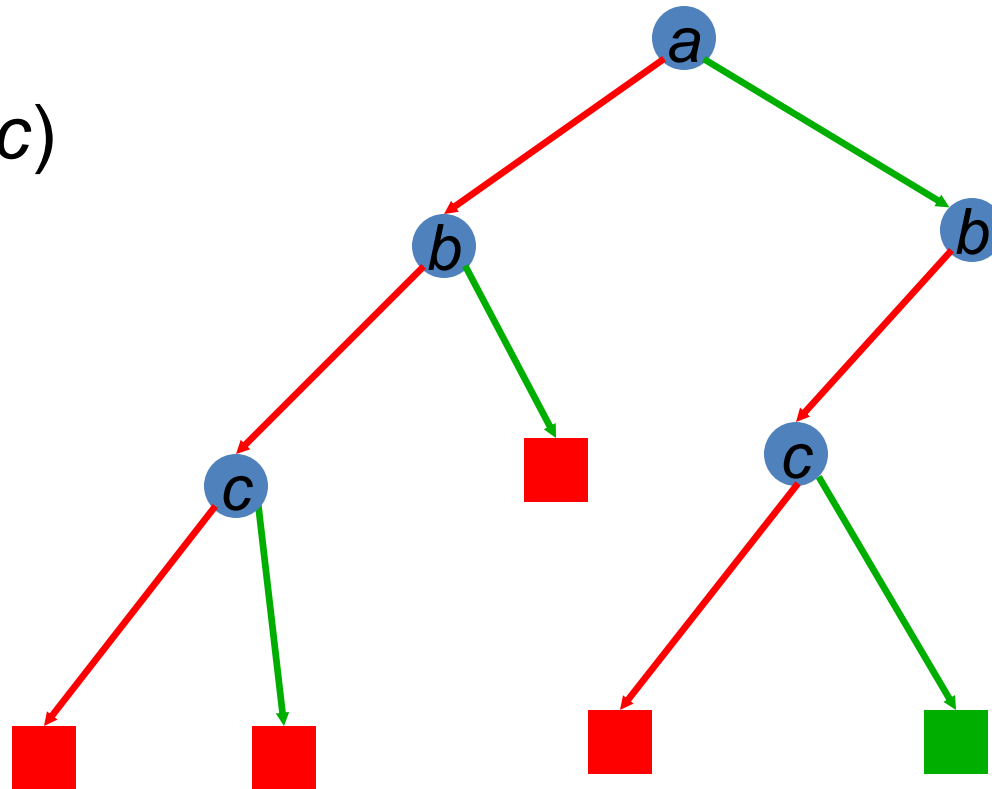
DPLL Version 1

$(a \vee b \vee c)$

$(a \vee \neg b)$

$(a \vee \neg c)$

$(\neg a \vee c)$



DPLL as Search

- Search Space?
- Algorithm?

Improving DPLL

If literal L_1 is true, then clause $(L_1 \vee L_2 \vee \dots)$ is true

If clause C_1 is true, then $C_1 \wedge C_2 \wedge C_3 \wedge \dots$ has the same value as $C_2 \wedge C_3 \wedge \dots$

Therefore: Okay to delete clauses containing true literals!

If literal L_1 is false, then clause $(L_1 \vee L_2 \vee L_3 \vee \dots)$ has the same value as $(L_2 \vee L_3 \vee \dots)$

Therefore: Okay to shorten clauses containing false literals!

If literal L_1 is false, then clause (L_1) is false

Therefore: the empty clause means false!

DPLL version 2

```
dp11_2(F, literal){
  remove clauses containing literal
  if (F contains no clauses) return true;
  shorten clauses containing  $\neg$ literal
  if (F contains empty clause)
    return false;
  choose V in F;
  if (dp11_2(F,  $\neg$ V)) return true;
  return dp11_2(F, V);
}
```

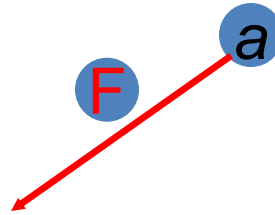

DPLL Version 2

$(F \vee b \vee c)$

$(F \vee \neg b)$

$(F \vee \neg c)$

$(T \vee c)$

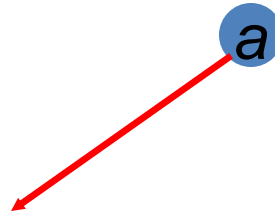


DPLL Version 2

$(b \vee c)$

$(\neg b)$

$(\neg c)$

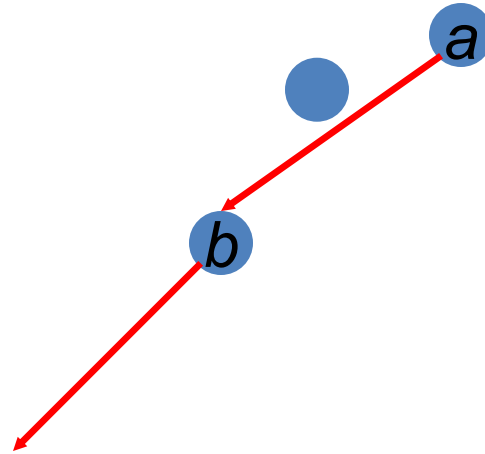


DPLL Version 2

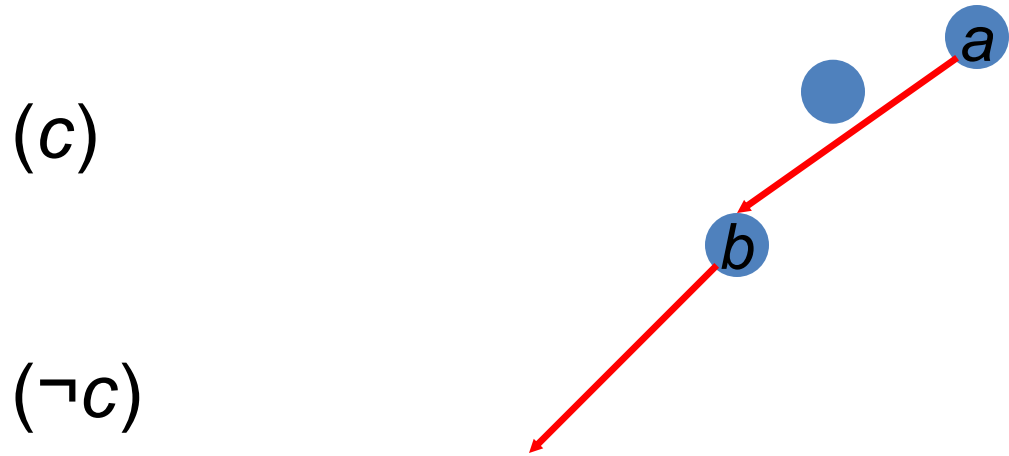
$(F \vee c)$

(T)

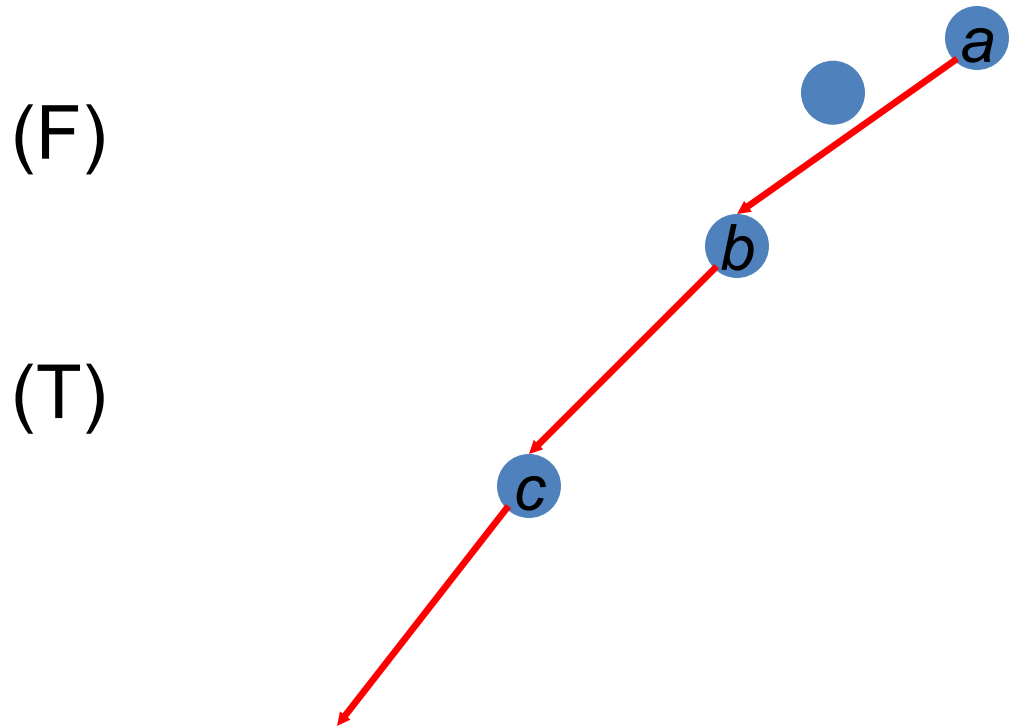
$(\neg c)$



DPLL Version 2

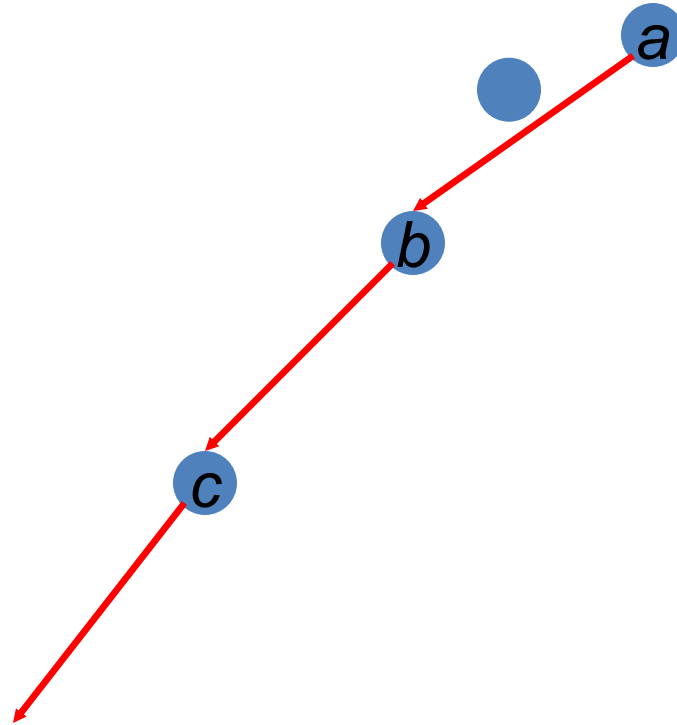


DPLL Version 2



DPLL Version 2

Empty clause!
()



Structure in Clauses

- Unit Literals (unit propagation)

A literal that appears in a singleton clause

$\{\{\neg b\ c\}\{\neg c\}\{a\ \neg b\ e\}\{d\ b\}\{e\ a\ \neg c\}\}$

Might as well set it true! And simplify

$\{\{\neg b\}\} \quad \{a\ \neg b\ e\}\{d\ b\}\}$
 $\{\{d\}\}$

- Pure Literals

– A symbol that always appears with same sign

– $\{\{a\ \neg b\ c\}\{\neg c\ d\ \neg e\}\{\neg a\ \neg b\ e\}\{d\ b\}\{e\ a\ \neg c\}\}$

Might as well set it true! And simplify

$\{\{a\ \neg b\ c\}\} \quad \{\neg a\ \neg b\ e\} \quad \{e\ a\ \neg c\}$

In Other Words

Formula $(L) \wedge C_2 \wedge C_3 \wedge \dots$ is only true when literal L is true

Therefore: Branch immediately on unit literals!

May view this as adding
constraint propagation
techniques into play

In Other Words

Formula $(L) \wedge C_2 \wedge C_3 \wedge \dots$ is only true when literal L is true

Therefore: Branch immediately on unit literals!

If literal L does not appear negated in formula F , then setting L true preserves satisfiability of F

Therefore: Branch immediately on pure literals!

May view this as adding
constraint propagation
techniques into play

DPLL (previous version)

Davis – Putnam – Loveland – Logemann

```
dp11(F, literal) {  
  remove clauses containing literal  
  if (F contains no clauses) return true;  
  shorten clauses containing  $\neg$ literal  
  if (F contains empty clause)  
    return false;  
  
  choose V in F;  
  if (dp11(F,  $\neg$ V)) return true;  
  return dp11(F, V);  
}
```

DPLL (for real!)

Davis – Putnam – Loveland – Logemann

```
dp11(F, literal) {  
  remove clauses containing literal  
  if (F contains no clauses) return true;  
  shorten clauses containing  $\neg$ literal  
  if (F contains empty clause)  
    return false;  
  if (F contains a unit or pure L)  
    return dp11(F, L);  
  choose V in F;  
  if (dp11(F,  $\neg$ V)) return true;  
  return dp11(F, V);  
}
```

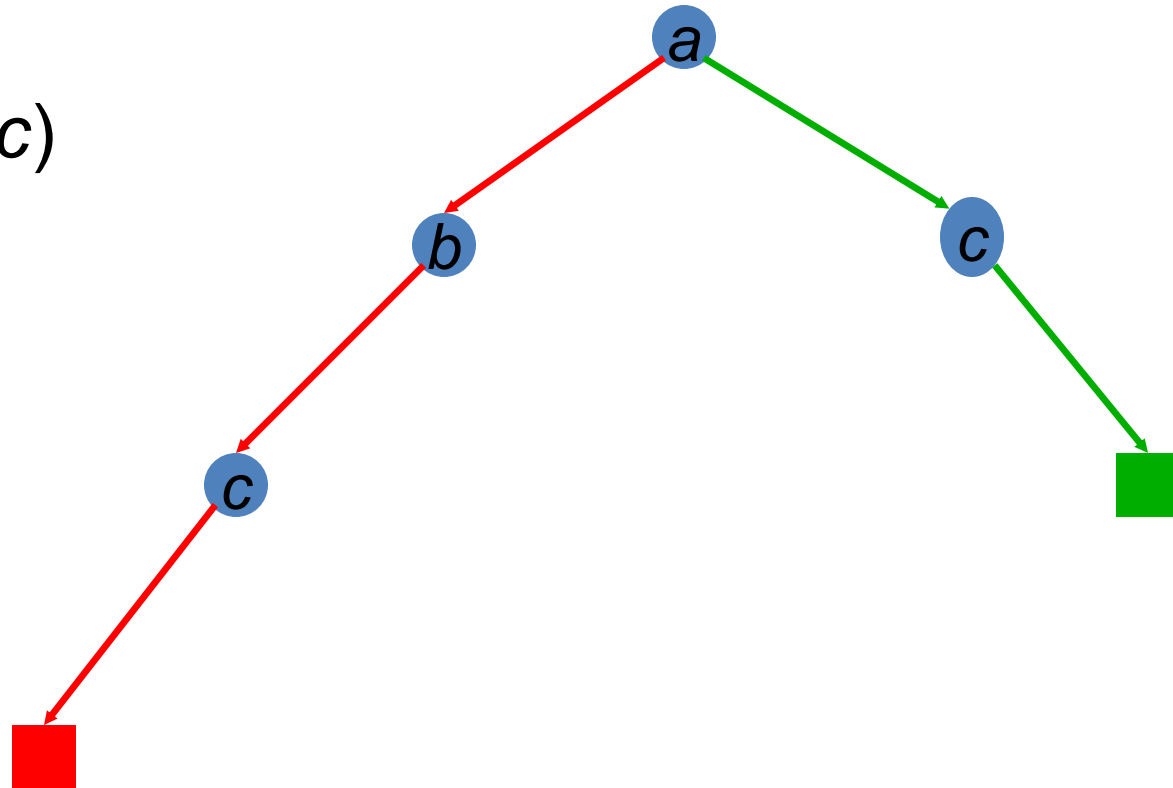
DPLL (for real)

$(a \vee b \vee c)$

$(a \vee \neg b)$

$(a \vee \neg c)$

$(\neg a \vee c)$



DPLL (for real!)

Davis – Putnam – Loveland – Logemann

```
dpll(F, literal){
  remove clauses containing literal
  if (F contains no clauses) return true;
  shorten clauses containing  $\neg$ literal
  if (F contains empty clause)
    return false;
  if (F contains a unit or pure L)
    return dpll(F, L);
  choose V in F;
  if (dpll(F,  $\neg$ V)) return true;
  return dpll(F, V);
}
```

Where could we use a heuristic to further improve performance?

Heuristic Search in DPLL

- Heuristics are used in DPLL to select a (non-unit, non-pure) proposition for branching
- Idea: identify a most constrained variable
 - Likely to create many unit clauses
- MOM's heuristic:
 - Most occurrences in clauses of minimum length

GSAT

- **Local** search (Hill Climbing + Random Walk) over space of **complete** truth assignments
 - With prob p : flip **any** variable in any unsatisfied clause
 - With prob $(1-p)$: flip **best** variable in any unsat clause
 - best = one which minimizes #unsatisfied clauses
- SAT encodings of N-Queens, scheduling
- Best algorithm for random K-SAT
 - Best DPLL: 700 variables
 - Walksat: 100,000 variables

Refining Greedy Random Walk

- Each flip
 - **makes** some false clauses become true
 - **breaks** some true clauses, that become false
- Suppose $s1 \rightarrow s2$ by flipping x . Then:
 - $\#unsat(s2) = \#unsat(s1) - make(s1,x) + break(s1,x)$
- **Idea 1:** if a choice breaks nothing, it is very likely to be a good move
- **Idea 2:** near the solution, only the break count matters
 - the make count is usually 1

Walksat

```
state = random truth assignment;
while ! GoalTest(state) do
  clause := random member { C | C is false in state };
  for each x in clause do compute break[x];
  if exists x with break[x]=0 then var := x;
  else
    with probability p do
      var := random member { x | x is in clause };
    else (probability 1-p)
      var := argminx { break[x] | x is in clause };
    endif
  state[var] := 1 - state[var];
end
return state;
```

**Put everything inside of a restart loop.
Parameters: p, max_flips, max_runs**

Advantages of WalkSAT over GSAT

- WalkSat guaranteed to make at least 1 false clause true (in random walk also)
- Number of evaluations small per move
 - does not iterate over all variables
 - only variables in the sampled clause