## An Introduction to Neural Nets & Deep Learning

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# The human brain is extremely good at classifying images

Can we develop classification methods by emulating the brain?

#### Brain Computer: What is it?





Biological Neuron - The simple "arithmetic computing" element



#### Brains

 $10^{11}$  neurons of  $\,>20$  types,  $10^{14}$  synapses, 1ms–10ms cycle time Signals are noisy "spike trains" of electrical potential



#### **Biological Neurons**

- Soma or body cell is a large, round central body in which almost all the logical functions of the neuron are realized.
- 2. The axon (output), is a nerve fibre attached to the soma which can serve as a final output channel of the neuron. An axon is usually highly branched.
- **3.** The dendrites (inputs)- represent a highly branching tree of fibres. These long irregularly shaped nerve fibres (processes) are attached to the soma.
- **4. Synapses** are specialized contacts on a neuron which are the termination points for the axons from other neurons.



#### Neurons communicate via spikes



Output spike roughly dependent on whether sum of all inputs reaches a threshold

#### Neurons as "Threshold Units"

- Artificial neuron:
  - m binary inputs (-1 or 1), 1 output (-1 or 1)
  - Synaptic weights w<sub>ii</sub>
  - Threshold  $\mu_{i}$



#### "Perceptrons" for Classification

- Fancy name for a type of layered "feed-forward" networks (no loops)
- Uses artificial neurons ("units") with binary inputs and outputs

Multilayer

Single-layer





#### Perceptrons and Classification

- Consider a single-layer perceptron
  - Weighted sum forms a *linear hyperplane*

$$\sum_{i} w_{ji} u_{j} - \mu_{i} = 0$$

- Everything on one side of this hyperplane is in class 1 (output = +1) and everything on other side is class 2 (output = -1)
- Any function that is <u>linearly separable</u> can be computed by a perceptron

#### Linear Separability

• Example: AND is linearly separable



v = 1 iff  $u_1 + u_2 - 1.5 > 0$ 

#### Similarly for OR and NOT

#### What about the XOR function?





## Can a perceptron separate the +1 outputs from the -1 outputs?

#### Linear Inseparability

- Perceptron with threshold units fails if classification task is not linearly separable
  - Example: XOR
  - No single line can separate the "yes" (+1) outputs from the "no" (-1) outputs!

Minsky and Papert's book showing such negative results put a damper on neural networks research for over a decade!



How do we deal with linear inseparability?

#### Idea 1: Multilayer Perceptrons

- Removes limitations of single-layer networks
  - Can solve XOR
- Example: Two-layer perceptron that computes XOR



• Output is +1 if and only if  $x + y - 2\Theta(x + y - 1.5) - 0.5 > 0$ 









## Activation functions

Non-linearities needed to learn complex (non-linear) representations of data, otherwise the NN would be just a linear function  $W_1W_2x = Wx$ 



http://cs231n.github.io/assets/nn1/layer\_sizes.jpeg

More layers and neurons can approximate more complex functions

Full list: https://en.wikipedia.org/wiki/Activation\_function

#### **Activation Functions**



## Activation: Sigmoid



Takes a real-valued number and "squashes" it into range between 0 and 1.

$$R^n \rightarrow [0,1]$$

- + Nice interpretation as the firing rate of a neuron
  - 0 = not firing at all
  - 1 = fully firing
- Sigmoid neurons saturate and kill gradients, thus NN will barely learn
  - when the neuron's activation are 0 or 1 (saturate)
    - □ gradient at these regions almost zero
    - □ almost no signal will flow to its weights
    - ☐ if initial weights are too large then most neurons would saturate

## Activation: Tanh



Takes a real-valued number and "squashes" it into range between -1 and 1.

$$R^n \rightarrow [-1,1]$$

- Like sigmoid, tanh neurons saturate
- Unlike sigmoid, output is zero-centered
- Tanh is a scaled sigmoid: tanh(x) = 2sigm(2x) 1

## Activation: ReLU



Takes a real-valued number and thresholds it at zero f(x) = max(0, x)

$$R^n \to R^n_+$$

Most Deep Networks use ReLU nowadays

#### Trains much faster

- accelerates the convergence of SGD
- due to linear, non-saturating form
- □ Less expensive operations
  - compared to sigmoid/tanh (exponentials etc.)
  - implemented by simply thresholding a matrix at zero
- □ More **expressive**
- Prevents the gradient vanishing problem

#### Example Application

• Handwriting Digit Recognition



#### Handwriting Digit Recognition

Input



Ink  $\rightarrow 1$ No ink  $\rightarrow 0$  Output



Each dimension represents the confidence of a digit.

#### Example Application

Handwriting Digit Recognition



In deep learning, the function f is represented by neural network

#### Element of Neural Network

**Neuron**  $f: \mathbb{R}^K \to \mathbb{R}$ 





Deep means many hidden layers

#### Example of Neural Network



#### Example of Neural Network



#### Example of Neural Network



 $f: \mathbb{R}^2 \to \mathbb{R}^2 \qquad f\left(\begin{bmatrix} 1\\-1 \end{bmatrix}\right) = \begin{bmatrix} 0.62\\0.83 \end{bmatrix} \quad f\left(\begin{bmatrix} 0\\0 \end{bmatrix}\right) = \begin{bmatrix} 0.51\\0.85 \end{bmatrix}$ 

Different parameters define different function

#### Matrix Operation



#### Neural Network



#### Neural Network



 $\mathbf{y} = f(\mathbf{x})$ 

Using parallel computing techniques to speed up matrix operation

$$= \sigma(W^{L} \cdots \sigma(W^{2} \sigma(W^{1} x + b^{1}) + b^{2}) \cdots + b^{L})$$

#### Softmax

Softmax layer as the output layer

#### **Ordinary Layer**



In general, the output of network can be any value.

May not be easy to interpret

#### Softmax

• Softmax layer as the output layer

Softmax Layer




#### How to set network parameters $\theta = \{W^1, b^1, W^2, b^2, \cdots W^L, b^L\}$



#### Training Data

• Preparing training data: images and their labels



Using the training data to find the network parameters.



target



Cost can be Euclidean distance or cross entropy of the network output and target

Cost

### Total Cost

For all training data ...



Total Cost:

$$C(\theta) = \sum_{r=1}^{R} L^{r}(\theta)$$

How bad the network parameters  $\theta$  is on this task

Find the network parameters  $\theta^*$  that minimize this value

# Gradient Descent

Assume there are only two parameters  $w_1$  and  $w_2$  in a network.

$$\theta = \{w_1, w_2\}$$

Randomly pick a starting point  $\theta^0$ 

Compute the negative gradient at  $\theta^0$ 

$$\rightarrow -\nabla C(\theta^0)$$

Times the learning rate  $\eta$ 





#### Gradient Descent



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#### Local Minima

С

Gradient descent never guarantee global minima



Different initial point  $\theta^0$ 

$$\blacksquare$$

Reach different minima, so different results

<sup>3</sup>Who is Afraid of Non-Convex Loss Functions? <u>http://videolectures.net/eml07</u> <u>lecun\_wia/</u>

#### Besides local minima .....



#### Mini-batch



- $\succ$  Randomly initialize  $\theta^0$
- Pick the 1<sup>st</sup> batch
  C = L<sup>1</sup> + L<sup>31</sup> + ···
  θ<sup>1</sup> ← θ<sup>0</sup> η∇C(θ<sup>0</sup>)
  Pick the 2<sup>nd</sup> batch
  C = L<sup>2</sup> + L<sup>16</sup> + ···
  θ<sup>2</sup> ← θ<sup>1</sup> η∇C(θ<sup>1</sup>)
  :

C is different each time when we update parameters!

# SGD vs. GD

• Deterministic gradient method [Cauchy, 1847]:



• Stochastic gradient method [Robbins & Monro, 1951]:





Stochastic will be superior for low-accuracy/time situations.



- A network can have millions of parameters.
  - Backpropagation is the way to compute the gradients efficiently
  - Ref: http://speech.ee.ntu.edu.tw/~tlkagk/courses/MLDS\_201
     5\_2/Lecture/DNN%20backprop.ecm.mp4/index.html
- Many toolkits can compute the gradients automatically

# theano





Ref:

http://speech.ee.ntu.edu.tw/~tlkagk/courses/MLDS\_2015\_2/Lec ture/Theano%20DNN.ecm.mp4/index.html

- If we choose a differentiable loss, then the the whole function will be differentiable with respect to all parameters.
- Because of non-linear activations whose combination is not convex, the overall learning problem is not convex.
- What does (stochastic) (sub)gradient descent do with nonconvex functions? It finds a local minimum.
- To calculate gradients, we need to use the chain rule from calculus.
- Special name for (S)GD with chain rule invocations: backpropagation.

For every node in the computation graph, we wish to calculate the first derivative of  $L_n$  with respect to that node. For any node a, let:

$$\bar{a} = \frac{\partial L_n}{\partial a}$$

Base case:

$$\bar{L_n} = \frac{\partial L_n}{\partial L_n} = 1$$

For every node in the computation graph, we wish to calculate the first derivative of  $L_n$  with respect to that node. For any node a, let:

$$\bar{a} = \frac{\partial L_n}{\partial a}$$

After working forwards through the computation graph to obtain the loss  $L_n$ , we work *backwards* through the computation graph, using the chain rule to calculate  $\bar{a}$  for every node a, making use of the work already done for nodes that depend on a.

$$\begin{split} \frac{\partial L_n}{\partial a} &= \sum_{b:a \to b} \frac{\partial L_n}{\partial b} \cdot \frac{\partial b}{\partial a} \\ \bar{a} &= \sum_{b:a \to b} \bar{b} \cdot \frac{\partial b}{\partial a} \\ &= \sum_{b:a \to b} \bar{b} \cdot \begin{cases} 1 & \text{if } b = a + c \text{ for some } c \\ c & \text{if } b = a \cdot c \text{ for some } c \\ 1 - b^2 & \text{if } b = \tanh(a) \end{cases} \end{split}$$

Pointwise ("Hadamard") product for vectors in  $\mathbb{R}^n$ :

$$\mathbf{a} \odot \mathbf{b} = \begin{bmatrix} \mathbf{a}[1] \cdot \mathbf{b}[1] \\ \mathbf{a}[2] \cdot \mathbf{b}[2] \\ \vdots \\ \mathbf{a}[n] \cdot \mathbf{b}[n] \end{bmatrix}$$

$$\begin{split} \mathbf{\bar{a}} &= \sum_{\mathbf{b}: \mathbf{a} \to \mathbf{b}} \sum_{i=1}^{|\mathbf{b}|} \mathbf{\bar{b}}[i] \cdot \frac{\partial \mathbf{b}[i]}{\partial \mathbf{a}} \\ &= \sum_{\mathbf{b}: \mathbf{a} \to \mathbf{b}} \begin{cases} \mathbf{\bar{b}} & \text{if } \mathbf{b} = \mathbf{a} + \mathbf{c} \text{ for some } \mathbf{c} \\ \mathbf{\bar{b}} \odot \mathbf{c} & \text{if } \mathbf{b} = \mathbf{a} \odot \mathbf{c} \text{ for some } \mathbf{c} \\ \mathbf{\bar{b}} \odot (\mathbf{1} - \mathbf{b} \odot \mathbf{b}) & \text{if } \mathbf{b} = \tanh(\mathbf{a}) \end{cases} \end{split}$$



Intermediate nodes are de-anonymized, to make notation easier.



 $\frac{\partial L_n}{\partial L_n} = 1$ 



The form of  $\bar{g}$  will be loss-function specific (e.g.,  $-2(y_n - g)$  for squared loss).



Sum.



Product.



Hyperbolic tangent.



Sum.



Product.

Part II: Why Deep?

# Deeper is Better?

Layer X Size	Word Error Rate (%)	
1 X 2k	24.2	
2 X 2k	20.4	
3 X 2k	18.4	
4 X 2k	17.8	
5 X 2k	17.2	
7 X 2k	17.1	

Not surprised, more parameters, better performance

Seide, Frank, Gang Li, and Dong Yu. "Conversational Speech Transcription Using Context-Dependent Deep Neural Networks." *Interspeech*. 2011.

# Universality Theorem

Any continuous function f

 $f: \mathbb{R}^N \to \mathbb{R}^M$ 

Can be realized by a network with one hidden layer

(given **enough** hidden neurons)



Reference for the reason: http://neuralnetworksandde eplearning.com/chap4.html

Why "Deep" neural network not "Fat" neural network?

#### Fat + Short v.s. Thin + Tall



# Fat + Short v.s. Thin + Tall

Layer X Size	Word Error Rate (%)	Layer X Size	Word Error Rate (%)
1 X 2k	24.2		
2 X 2k	20.4		
3 X 2k	18.4		
4 X 2k	17.8		
5 X 2k	17.2 🔶	→ 1 X 3772	22.5
7 X 2k	17.1 🔶	🔶 1 X 4634	22.6
		1 X 16k	22.1

Seide, Frank, Gang Li, and Dong Yu. "Conversational Speech Transcription Using Context-Dependent Deep Neural Networks." *Interspeech*. 2011.

## Why Deep?

#### • Deep $\rightarrow$ Modularization



Why Deep?

Each basic classifier can have sufficient training examples.

• Deep  $\rightarrow$  Modularization





Part III: Tips for Training DNN

# Recipe for Learning



http://www.gizmodo.com.au/2015/04/the-basic-recipe-for-machine-learning-explained-in-a-single-powerpoint-slide/

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http://www.gizmodo.com.au/2015/04/the-basic-recipe-for-machine-learning-explained-in-a-single-powerpoint-slide/
# Recipe for Learning

#### Modify the Network

New activation functions, for example, ReLU or Maxout

#### **Better optimization Strategy**

• Adaptive learning rates

#### **Prevent Overfitting**

• Dropout

Only use this approach when you already obtained good results on the training data.

Part III: Tips for Training DNN Dropout



- > Each time before computing the gradients
  - Each neuron has p% to dropout



- Each time before computing the gradients
  - Each neuron has p% to dropout

The structure of the network is changed.

Using the new network for training

For each mini-batch, we resample the dropout neurons

# Dropout

#### **Testing:**



#### No dropout

- If the dropout rate at training is p%, all the weights times (1-p)%
- Assume that the dropout rate is 50%.
  If a weight w = 1 by training, set w = 0.5 for testing.

# Dropout - Intuitive Reason



- When teams up, if everyone expect the partner will do the work, nothing will be done finally.
- However, if you know your partner will dropout, you will do better.
- When testing, no one dropout actually, so obtaining good results eventually.

Part IV: Convolutional Neural Nets

# Traditional ML vs. Deep Learning

Most machine learning methods work well because of human-designed representations and input features

ML becomes just optimizing weights to best make a final prediction



Feature	NER
Current Word	1
Previous Word	1
Next Word	1
Current Word Character n-gram	all
Current POS Tag	1
Surrounding POS Tag Sequence	1
Current Word Shape	1
Surrounding Word Shape Sequence	1
Presence of Word in Left Window	size 4
Presence of Word in Right Window	size 4

# What is Deep Learning (DL) ?

A machine learning subfield of learning **representations** of data. Exceptional effective at **learning patterns**.

Deep learning algorithms attempt to learn (multiple levels of) representation by using a hierarchy of multiple layers

If you provide the system **tons of information**, it begins to understand it and respond in useful ways.



https://www.xenonstack.com/blog/static/public/uploads/media/machine-learning-vs-deep-learning.png

# Why is DL useful?

- Manually designed features are often over-specified, incomplete and take a long time to design and validate
- Learned Features are easy to adapt, fast to learn
- Deep learning provides a very flexible, (almost?) universal, learnable framework for representing world, visual and linguistic information.
- $\circ~$  Can learn both unsupervised and supervised
- Effective end-to-end joint system learning
- Utilize large amounts of training data



In ~2010 DL started outperforming other ML techniques first in speech and vision, then NLP

#### Feature Learning









#### Basic Concept of CNN

- Convolutional neural networks
  - Signal, image, video



#### Architecture of LeNet

- Convolutional layers
- Sub-sampling layers
- Fully-connected layers



## Convolution



Convolved Feature

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

5x5 input. convolved feature/

3x3 filter/kernel/feature detector. 3x3

### Multiple filters



Original image

Operation	Filter	Convolved Image
Identity	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
	$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	
Edge detection	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	
Sharpen	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
Box blur (normalized)	$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	

### Features at successive convolutional layers



Corners and other edge color conjunctions in Layer 2

### Features at successive convolutional layers



More complex invariances than Layer 2. Similar textures e.g. mesh patterns (R1C1); Text (R2C4).

### Features at successive convolutional layers



Significant variation, more class specific. Dog faces (R1C1); Bird legs (R4C2).

Entire objects with significant pose variation. Keyboards (R1C1); dogs (R4).

## Who decides these features?

The network itself while training learns the filter weights and bias terms.



Evolution of randomly chosen subset of model features at training epochs 1,2,5,10,20,30,40,64.

### Max pooling

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

max pool with 2x2 filters and stride 2

6	8		
3	4		

#### **CNN** architecture



# **Object Recognition**





- airplane automobile bird
- cat deer
- dog
- frog horse ship
- truck

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Network	Error	Layers
AlexNet	16.0%	8
ZFNet	11.2%	8
VGGNet	7.3%	19
GoogLeNet	6.7%	22
MS ResNet	3.6%	152!!