## Stacks, Queues, Linked Lists \& Recursion, Amortized Analysis

(1) Describe a recursive algorithm to compute the integer part of the base-two logarithm of $n$ using only addition and integer division.
(2) Suppose you are given an $n$-element array $A$ containing distinct integers that are listed in increasing order. Given a number $k$, describe a recursive algorithm the find two integers in $A$ that sum to $k$, if such a pair exists. What is the running time of your algorithm?
(3) Describe a linear time algorithm to reverse a singly-linked list $L$ so that the ordering of the nodes becomes opposite of what it was before.
(4) Describe in pseudo-code how to implement the stack ADT using two queues. What is the running time of push(), and $\operatorname{pop}()$ methods in this case? (Note: there are multiple solutions to this. Think of a solution where push() is constant time, and another solution where $\operatorname{pop}()$ is constant time).
(5) A (singly linked) circular list is a collection $C$ of $n$ positions such that each has a next variable and following next links starting from any position can visit all positions in $C$. Describe how to perform insertBefore $(p, e)$ and $\operatorname{insertAfter}(p, e)$ for position $p$ and element $e$ in such a scheme. What are the running times of these operations? Can you do both operations in constant time?
(6) Consider the problem of storing a very large counter. Say, we use an array $A$ where $i^{\text {th }}$ bit of the count is stored $A[i]$. Suppose the counter had to count a total of $N$ events. What is the total running time of counting up to $N$ using a such a counter? (Hint: think amortized analysis).
(7) [Challenge Problem] A singly linked list $L$ has a cycle if the last position of the list, instead of having a null next pointer, points to some previous position in the list. Develop a method for checking if $L$ has a cycle.

