

## ASYMPTOTIC ANALYSIS

- (1) Order the following functions in the increasing order of their asymptotic growth rate:  
(a)  $4n \log n + 2n$ , (b)  $2^{10}$ , (c)  $2^{\log n}$ , (d)  $3n + 100 \log n$ , (e)  $4n$ , (f)  $2^n$ , (g)  $n^2 - 10n$ , (h)  $n^3$ , (i)  $n \log n$
- (2) Show that  $O(\max\{f(n), g(n)\})$  is  $O(f(n) + g(n))$ .
- (3) Show that if  $d(n)$  is  $O(f(n))$  and  $e(n)$  is  $O(g(n))$  then  $d(n) - e(n)$  is not necessarily  $O(f(n) - g(n))$ .
- (4) Show that  $n^2$  is not  $o(n^2)$ .
- (5) Suppose that each row of an  $n \times n$  array  $A$  consists of 1s and 0s such that, in any row of  $A$ , all the 1s come before any 0s in that row. Assuming  $A$  is already in memory, describe a method running in  $\Theta(n)$  time for finding the row of  $A$  that contains the most 1s. Explain why your algorithm is  $\Theta(n)$ .
- (6) Describe in pseudo-code a method for multiplying an  $n \times m$  matrix  $A$  and an  $m \times p$  matrix  $B$ . Recall that the product  $C = AB$  is defined so that  $C[i][j] = \sum_{k=1}^m A[i][k] \cdot B[k][j]$ . What is the running time of your method?
- (7) [Challenge Problem] Describe a method for finding both the minimum and maximum of  $n$  numbers using fewer than  $3n/2$  comparisons.