CSL860: Routing in the presence of faults
I semester, 2008-09

Minor I

September 5, 2008

Due: **Handwritten**. Slipped into my office before **11AM** on **Monday, 8th September 2008**. **Latexed**. Printed out and handed to me at the beginning to class on **Tuesday, 9th September 2008**.

Examination notes: Please read these carefully before beginning.. Please answer in whatever detail you feel necessary. Any statement that requires proof should be identified as such. Either prove such a statement or say that you will not be proving it. Use your judgment on what to take without proof. If something is crucial to the overall argument it must be proved. You are free to consult with your friends as long as you mention this in the body of your answer sheet. I will scale your grade down by 10 percent for this. You are also allowed to look up the internet or other source for an additional penalty of 10 percent. You must mention what web site or other source you consulted. Failing to mention either of these two will lead to harsher penalties. Also note that the deadlines for handing in the papers are hard, they will not be relaxed under any circumstance. I would suggest you complete as much of the paper as you can by the time given and turn it in. If after handing in a handwritten exam you decide you want more time and are willing to latex your exam to get that extra time, I will gladly accept it.

Problem 1. In Lecture 1 we considered the problem of routing on the ring and showed that *Furthest-to-go* was an optimal strategy. Now let us consider the tie-breaking strategy *FCFS* i.e. the strategy in which the packet that arrives earlier (where "arrival" includes the possibility that it was generated at that node) gets preference for forwarding (with further ties being resolved by name as before). Show that for any value k it is possible to generate an instance on the n-node ring such that *FCFS* takes k more steps to complete than *Furthest-to-go*. What is the highest value of k you can achieve with an input instance that has a total of N packets in it?

Problem 2. Given a graph G and a number p s.t. $0 \leq p \leq 1$, let us denote by $G(p)$ the (random) subgraph of G obtained by retaining each edge of G with probability p. Let $\{G_n = (V_n, E_n)\}$ be a sequence of graphs with maximum degree Δ each of which is an α-expander i.e. it has edge expansion α for some constant value α (i.e. some value independent of n.) We assume that $|V_n| = n$.

Let A be such that $(\Delta e) \cdot A^{\alpha} < 1$ and let $1 - A \leq p_n \leq 1$ for each n. Show that for any $1/2 > c > 0$

$$\Pr(G_n(p_n) \text{ contains a component of size between } c|V_n| \text{ and } \frac{1}{2}|V_n|) \to 0,$$

as $n \to \infty$. (**Hint.** You may use without proof the fact that the number of connected subsets of size r of the vertex set of a graph of max degree Δ containing a given vertex is at most $(\Delta e)^r$.)

What conclusions can you draw from this theorem about (a) the size(s) of components in $G_n(p_n)$ and (b) the number of components in $G_n(p_n)$?

Problem 3. We continue with the scenario described above. Let us take $G_n(p_n)$ and apply a variant of the pruning algorithm described in class i.e.

> While there is a set with expansion less than $\epsilon\alpha$ left in $G_n(p_n)$, remove it and all edges associated with it.

Let us denote the graph left at the end of the pruning process by $H_n = (V_n^H, E_n^H)$.

3.1. What is the expansion of H_n? (**Hint**. Use Theorem 2.10 of Lecture 2.)

3.2. Prove that there is a constant c_1 such that $|V_n^H| \geq c_1 \cdot |V_n|$ as long as the condition on A given above holds.

Proceed by assuming that more than $(1 - c_n)|V_n|$ vertices have been pruned away. Show that the probability of this event is low.