Release date: June 7, 2020 Deadline: June 14, 2020: 23:00

Refer to recorded lectures 1-3 for this homework. Submission format: PDF is preferred. Photographs and scans are also acceptable, but it's your responsibility to ensure that they are clear and readable.

1. **[6 points]** Consider the following weighted version of non-preemptive bipartite matching in the vertex arrival setting. Each offline vertex i has a non-negative weight w_i, and these weights are provided in the beginning with the offline vertices. The weight of a matching is the sum of the weights of the matched offline vertices, and we want to compete with the weight of the maximum weight matching. The rest of the problem definition is the same: in each round, a new online vertex appears along with the edges incident on it, and we must match it irrevocably to some available neighbor or leave it unmatched. Consider the following algorithm.

   - For each offline vertex i, sample $X_i \sim U[0,1]$ independently. ($U[0,1]$ denotes the uniform distribution on $[0,1]$.)
   - For each online vertex j, if j has at least one unmatched neighbor, match j to that unmatched neighbor i which maximizes $w_i \cdot (1 - e^{X_i - 1})$.

   Prove that this algorithm is $(1-1/e)$-competitive. (Observe that when the weights w_i are all equal, this algorithm is same as the Karp-Vazirani-Vazirani algorithm. The analysis will almost follow the same footsteps, but be careful and spot all the differences. If the proof of some claim in your analysis is exactly the same as in the Karp-Vazirani-Vazirani analysis, you may say so and skip the proof.)

2. **[4 points]** In the third recorded lecture around the 48:00 timestamp (on the third page of the scanned notes), I wrote the following claim:

$$\mathbb{E}[|M|] \leq n\left(1 - \frac{1}{e}\right) + o(n),$$

   and left the details for you to figure out. Complete that proof. (Again, you don't have to reprove the claims already proven in the recorded lecture.)