COL 702

Homework I

Due on Aug. 19, 2019

1. Show that any undirected graph has at least two vertices of the same degree.

2. Let $G = (V, E)$ be an undirected graph. Which of the following relations is an equivalence relation? Give reasons.

   - Two vertices u and v in V are related if there is a cycle containing u and v.
   - Two edges e and f in E are related if there is a cycle containing e and f.

3. Let C be a cycle in a directed graph G. Show that at least one edge of C must be a back edge while performing DFS on G. Is it possible that more than one edges of C become back edges ? Give reasons.

4. Describe an efficient algorithm to find the second minimum shortest path between vertices u and v in a weighted graph without negative weights. The second minimum weight path must differ from the shortest path by at least one edge and may have the same weight as the shortest path.

5. Given a directed acyclic graph that has maximal path length k, design an efficient algorithm that partitions the vertices into $k+1$ sets such that there is no path between any pair of vertices in a set.

6. Given an undirected graph $G = (V, E)$, create an efficient algorithm to find a maximum size (i.e., number of vertices) subgraph such that the degree of every vertex in this subgraph is at least k (or show that no such subgraph exists).