Symmetry based 3D Reconstruction of Repeated Cylinders

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Abstract—**First, we describe how** $360°$**-rotational symmetry may be used for three dimensional reconstruction of repeated cylinders from a single perspective image. In our experiments, we consider translational and affine repetition of cylinders with vertical and random orientations. Later, we create a virtual camera configuration for retrieving pose and location of repeated cylinders. The combination of** $360°$**-rotational symmetry and camera center is used to identify two orthogonal planes called** *axis plane* **and** *orthogonal axis plane*. **These two planes are the basis for the proposed reconstruction framework and virtual camera configuration. Furthermore, we discuss possible extension of our method in vision tasks based on motion analysis.**

Fig. 1.   World Coordinate System (Left-hand)

I. Introduction

Computational symmetry and group theory has been extensively studied in computer vision [1]. It is widely used for image segmentation and retrieving three dimensional structure and pose and computing structure from motion [2][3][4][5]. Here, we use rotational symmetry for 3D reconstruction of repeated cylinders with random orientations from single perspective image. These cylinders may have translational and affine repetitions. Since a cylinder has $360°$-rotational symmetry around its axis, it is convenient to use vanishing points based visual metrology techniques.

Single view metrology techniques are mostly used for retrieving metric measurements and calibrating cameras [6][7][8][9]. These techniques use homology, a plane projective transformation, for scenes containing parallel lines and parallel planes [10][6][7]. In the proposed method, we blend and develop previous work on (i) vanishing points based reconstruction methods; (ii) symmetry of individual 3D object; (iii) configuration of repeated objects.

In this paper, we consider a prespective image of randomly placed cylinders on the ground plane. Any one cylinder is chosen as principal cylinder and rest are called auxiliary cylinders. The $360°$-rotational symmetry and location of camera projection center are used in finding two orthogonal planes. First, *axis plane* passes through camera center and axis of cylinder. It bisects cylinder in Euclidean space and image plane too, if we ignore the lens distortion. Second, *orthogonal axis plane* passes through cylinder's axis and is orthogonal to *axis plane*.

These two planes are the basis for 3D reconstruction framework and creating a virtual camera configuration, an alternate framework, for retrieving 3D-box (pose and location) of auxiliary cylinder with respect to principal cylinder. Furthermore, proposed fr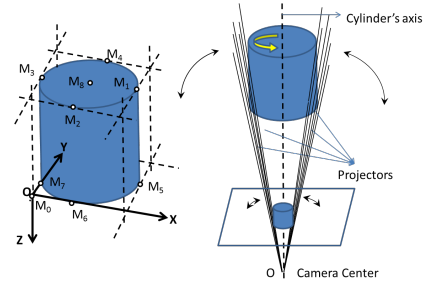amework can be extended for vision tasks involving multiple views such as motion analysis - structure from motion, motion segmentation and tracking.

Primarily, homology is used to find the distance between two parallel planes [6][10]. A set of three orthogonal pairs of parallel planes are required for retrieving 3D coordinates of a point. This is a three homology configuration. Since, a cylinder has $360°$-rotational symmetry around its axis, it requires only two homology to find 3D coordinates of a point on the top surface of a vertical cylinder. The 3D coordinates of the point give height and diameter. Similar techniques are used to find the measurements and position of arbitrarily placed auxiliary cylinder with respect to the principal cylinder.

Section II describes $360°$-rotational symmetry and two important orthogonal planes for a cylinder. A framework for computing measurement and location of principal and auxiliary cylinders is developed in section III. Detailed discussion on the virtual camera configuration for computing 3D-box of repeated cylinders is described in section IV. Results and conclusion with future work are discussed in Section V and VI, respectively.

II. Symmetry in the Geometry of a Cylinder

The chosen world coordinate system is shown in figure (1). Every cylinder has such coordinate system. However, measurements and location of all auxiliary cylinders are computed with respect to principal cylinder. Particularly, this coordinate system is useful in understanding rotational symmetry, finding two orthogonal planes, creating virtual camera configuration, as discussed in section IV, and computing point correspondences from image.
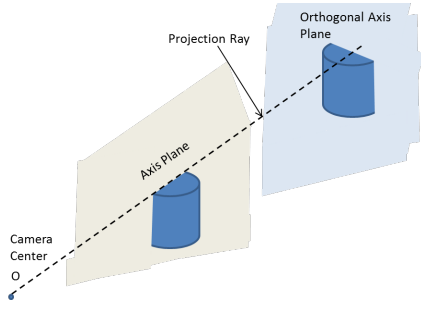
Fig. 2. Axis Plane and Orthogonal Axis Plane

A. Rotational Symmetry and Projection Center

All the projectors from camera center to a cylinder's axis form a plane. This is called an *axis plane*. The image of *axis plane* or image of cylinder's axis is a line segment that bisects the image of cylinder, if we ignore the lens distortion. Consequently, there is reflection symmetry in the image of every cylinder. This plane is identified due to $360°$-rotational symmetry in the shape of cylinder. Another important plane is *orthogonal axis plane*. It passes through cylinder's axis and is orthogonal to *axis plane*. These two planes are shown in figure (2).

"If a cylinder is rotated around its axis or is moved in world space such that the perpendicular distance of orthogonal axis plane from camera center is kept constant, the size and shape of its image will remain same".

This is an important conclusion which relates the motion of cylinder and its $360°$-rotational symmetry.

B. Point Correspondences

The *axis plane* and *orthogonal axis plane* are used to find corresponding points for each cylinder as shown in figure (1). Points m_i are image of points M_i, where $i = 0, \ldots, 8$. Image points m_1, m_3, m_5, and m_7 lie on *orthogonal axis plane*. The lines $\langle m_1, m_5 \rangle$ and $\langle m_3, m_7 \rangle$ are side-edges of image of cylinder and can be computed using edge detection techniques. Due to reflection symmetry, m_8 is mid point of m_1 and m_3 and image of visible circular surface is ellipse, if lens distortion is ignored. The line passing through m_8 and orthogonal to line $\langle m_1, m_3 \rangle$ intersects the conic at m_2 and m_4. We can use Bookstein's method for conic fitting [11]. Point M_8 is mid point of M_2 and M_4. Due to perspective distortion m_8 would not be mid point of m_2 and m_4. Point m_6 can be computed in the similar manner.

III. MEASUREMENT AND LOCATION OF CYLINDERS

A. Measurement of Principal Cylinder

The vanishing points of X and Z axes are computed as follows:

$$v_x = (m_1 \times m_3) \times (m_5 \times m_7), \ v_z = (m_3 \times m_7) \times (m_1 \times m_5)$$

Since M_8 is mid point of M_2 and M_4, 1D homography is used to compute the vanishing point of Y axis, v_y [10]. Point

v_0 is image of world origin.

$$v_0 = (m_6 \times v_x) \times (m_7 \times v_y)$$

To find measurements of principal cylinder, 3D coordinates of $M_1 = (X_1, \ Y_1, \ Z_1)$ should be computed. Point $M_5 = (X_1, \ Y_1, \ 0)$ is projection of M_1 on XY-plane. The camera projection matrix can be chosen as $P = \begin{pmatrix} v_x & v_y & v_z & v_0 \end{pmatrix}$ [10][7]. The projection of M_5 in terms of P is written as follows

$$\lambda_5 m_5 = X_1 v_x + Y_1 v_y + v_0$$

Taking scalar product of above equation with l_{xy} yields λ_5.

$$\lambda_5 = \frac{(v_0.l_{xy})}{(m_5.l_{xy})}$$

Similarly, m_1 can be expressed in terms of m_5.

$$\lambda_1 m_1 = X_1 v_x + Y_1 v_y + Z_1 v_z + v_0 = \lambda_5 m_5 + Z_1 v_z$$

Taking vector product of above equation with m_1 yields Z_1.

$$Z_1 = -\lambda_5 \frac{\|m_5 \times m_1\|}{\|v_z \times m_1\|}$$

Similarly, after taking scalar product of equation for m_5 with l_{zx} and l_{yz}, we get Y_1 and X_1, respectively.

$$Y_1 = \frac{\lambda_5 (m_5.l_{zx}) - (v_0.l_{zx})}{(v_y.l_{zx})}, \ X_1 = \frac{\lambda_5 (m_5.l_{yz}) - (v_0.l_{yz})}{(v_x.l_{yz})}$$

We assumed that principal cylinder is vertically oriented, though not necessary, and its height H and diameter D are known. The X_1, Y_1 and Z_1 are affine measurements. The constants $\alpha_x = \frac{X_1}{D}$, $\alpha_y = \frac{2Y_1}{D}$ and $\alpha_z = \frac{Z_1}{H}$ are fixed for the chosen world coordinate system [7]. So, the position of camera center, $T_C = (T_{CX} \ T_{CY} \ T_{CZ})^T$, is computed using vanishing points and these three constants [7][10].

B. Measurements and Pose of Auxiliary Cylinders

Position of auxiliary cylinder is computed with respect to principal cylinder. The key idea is not to use vanishing points of auxiliary cylinder. In this context, two different configurations are considered.

1) Vertical Orientation: Vertically oriented auxiliary cylinder may have translational or affine repetition. To find the measurement and location of auxiliary cylinder, 3D coordinates of $M_1' = (X_1', Y_1', Z_1')$, and $M_3' = (X_3', Y_3', Z_1')$ should be computed. $M_5' = (X_1', Y_1', 0)$ and $M_7' = (X_3', Y_3', 0)$ are projection of M_1' and M_3' on the XY-plane, respectively.

$$\lambda_5' m_5' = X_1' v_x + Y_1' v_y + v_0 \Rightarrow \lambda_5' = \frac{(v_0.l_{xy})}{(m_5'.l_{xy})}$$

Image point m_1' can be written in terms of m_5',

$$\lambda_1' m_1' = Z_1' v_z + \lambda_5' m_5'$$

Taking scalar product of above equation with m_1' yields Z_1'.

$$Z_1' = -\lambda_5' \frac{\|m_5' \times m_1'\|}{\|v_z \times m_1'\|}$$

Using λ_5' in the equation for m_5', we get expressions for X_1' and Y_1',

$$X_1' = \frac{\lambda_5'(m_5'.l_{yz}) - (v_0.l_{yz})}{(v_x.l_{yz})}, \ Y_1' = \frac{\lambda_5'(m_5'.l_{zx}) - (v_0.l_{zx})}{(v_y.l_{zx})}$$

In the similar fashion, we can solve equations for m_3' and m_7' and get expressions for X_3' and Y_3'.

$$\lambda_7'm_7' = X_3'v_x + Y_3'v_y + v_0 \Rightarrow \lambda_7' = \frac{(v_0.l_{xy})}{(m_7'.l_{xy})}$$

$$Y_3' = \frac{\lambda_7'(m_7'.l_{zx}) - (v_0.l_{zx})}{(v_z.l_{zx})}, \ X_3' = \frac{\lambda_7'(m_7'.l_{yz}) - (v_0.l_{yz})}{(v_x.l_{yz})}$$

2) Random Orientation: In this configuration cylinders are placed horizontally, as shown in figure (6). It is another case of affine repetition. The XY-plane is a tangent plane to the auxiliary cylinder and touches in a line segment. One end point of this segment is $M_9' = (X_{10}', Y_{10}', 0)$. The diameter through M_9' intersects the conic at $M_{10}' = (X_{10}', Y_{10}', Z_{10}')$. Another hypothetical plane, which is parallel to XY-plane, touches cylinder in a line segment $\langle M_{10}', M_{11}' \rangle$. The coordinates of point M_{11}' are $(X_{11}', Y_{11}', Z_{10}')$. To find the measurement and location of auxiliary cylinder, 3D coordinates of M_{10}' and M_{11}' should be computed.

The line $v_z \times m_8'$ intersects the visible conic at points m_9' and m_{10}'. In the similar fashion, image coordinates of m_{11}' can be computed. Once, image coordinates of m_9', m_{10}' and m_{11}' are computed, the expressions for λ_{10}, X_{10}', Y_{10}' and Z_{10}' are worked out as before,

$$\lambda_9'm_9' = X_{10}'v_x + Y_{10}'v_y + v_0 \Rightarrow \lambda_9' = \frac{(v_0.l_{xy})}{(m_9'.l_{xy})}$$

$$X_{10}' = \frac{\lambda_9'(m_9'.l_{yz}) - (v_0.l_{yz})}{(v_x.l_{yz})}, \ Y_{10}' = \frac{\lambda_9'(m_9'.l_{zx}) - (v_0.l_{zx})}{(v_y.l_{zx})}$$

$$\lambda_{10}'m_{10}' = Z_{10}'v_z + \lambda_9'm_9' \Rightarrow Z_{10}' = -\lambda_9'\frac{||m_9' \times m_{10}'||}{||v_z \times m_{10}'||}$$

We need to use Z_{10}' to compute expression for X_{11}' and Y_{11}'.

$$\lambda_{11}'m_{11}' = X_{11}'v_x + Y_{11}'v_y + Z_{10}'v_z + v_0$$

$$\lambda_{11}' = \frac{Z_{10}'(v_z.l_{xy}) + (v_0.l_{xy})}{(m_{11}'.l_{xy})}$$

$$X_{11}' = \frac{\lambda_{11}'(m_{11}'.l_{yz}) - (v_0.l_{yz})}{(v_x.l_{yz})}$$

$$Y_{11}' = \frac{\lambda_{11}'(m_{11}'.l_{zx}) - (v_0.l_{zx})}{(v_y.l_{zx})}$$

IV. VIRTUAL CAMERA CONFIGURATION

The proposed virtual camera configuration framework corroborates the previous discussion and provide an alternate method to compute 3D-box of auxiliary cylinders. *A single perspective image of repeated cylinders can be considered as multiple images of one cylinder.* This is called isometry property. *Axis plane* and *orthogonal axis plane* are dependent on camera center. Virtual camera for auxiliary cylinder is placed such that *virtual axis plane* and *virtual orthogonal axis*
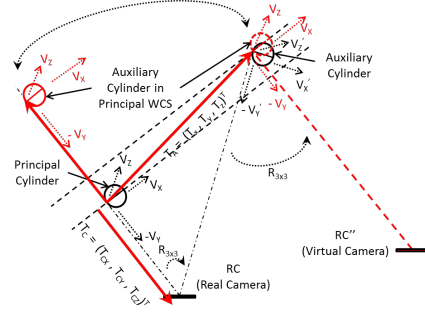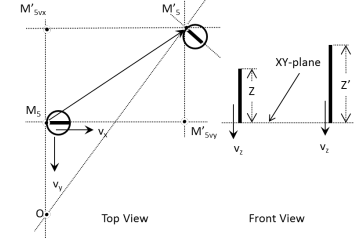

Fig. 3.   Virtual Camera Configuration

Fig. 4.   Location of Affinely Repeated Cylinder

plane are parallel to *axis plane* and *orthogonal axis plane* of principal cylinder, respectively, as shown in figure (3). This configuration of planes can be used to find the position of auxiliary cylinder with respect to principal cylinder.

A. Vertical Orientation

1) Translational Repetition: The position of auxiliary cylinder with respect to principal cylinder is represented by 3-vector $T' = \begin{pmatrix} X' & Y' & 0 \end{pmatrix}^T$, as shown in the figure (4) (top view). If $M_5 = \begin{pmatrix} X_5 & Y_5 & 0 \end{pmatrix}$ and $M_5' = \begin{pmatrix} X_5' & Y_5' & 0 \end{pmatrix}$ is a pair of corresponding point, length of line segments $\langle M_5', M_{5vx}' \rangle$ and $\langle M_5', M_{5vy}' \rangle$ are X and Y coordinates of vector T'. Points $M_{5vx}' = \begin{pmatrix} X_5 + X' & Y_5 & 0 \end{pmatrix}$ and $M_{5vy}' = \begin{pmatrix} X_5 & Y_5 + Y' & 0 \end{pmatrix}$ are projection of point M_5' on the planes which pass through point M_5 and are parallel to YZ and ZX planes, respectively.

$$m_{5vx}' = (m_5 \times v_y) \times (m_5' \times v_x)$$

$$m_{5vy}' = (m_5 \times v_x) \times (m_5' \times v_y)$$

Once image coordinates m_{5vx}' and m_{5vy}' are computed, X' and Y' can be computed by using similar calculus as discussed in section III.

2) Affine Repetition: In case of affine repetition, 3-vector $T' = \begin{pmatrix} X' & Y' & Z' \end{pmatrix}^T$ has non-zero Z coordinate, as shown in figure (4) (Front View). The X' and Y' coordinates are computed same as for translational repetition and expression for Z' is

$$Z' = \frac{||\lambda_1'(m_1' \times l_{xy}) - (v_0 \times l_{xy})||}{||v_z \times l_{xy}||}$$

where

$$\lambda_1' = \frac{||(X_5 + X')(v_x \times l_{yz}) + (v_0 \times l_{yz})||}{||m_1' \times l_{yz}||}$$

B. Random orientation

In this case, pose along with position is required for computing $3D$ box of auxiliary cylinder with respect to principal cylinder. The vector T' is computed as per section III-B2. Pose, which is rotation transformation, is computed using set of vanishing points of both cylinders. Geometry is shown in the figure (3).

$$\begin{pmatrix} v'_x & v'_y & v'_z \end{pmatrix} = R_{3\times 3} \begin{pmatrix} v_x & v_y & v_z \end{pmatrix}$$

Given metric measurements of principal cylinder (distances along X, Y and Z axes), $R_{3\times 3}$ can be computed [9][8].

V. Experimental Results

In our experiment, we considered two real images (figure (5) and (6)). Table I and II display the measurements (centimeter) of cylinders computed using methods discussed in section III-B1 and III-B2, respectively. The deviation of experimental measurements from scene measurements is acceptable. Those can be further improved by using efficient image processing techniques for finding point correspondences and computing vanishing points. There is a possibility of further improvement by employing uncertainly analysis as discussed in [7].

TABLE I
EXPERIMENTAL RESULTS FOR FIGURE (5)

Cylinder	Orientation	Type	Scene	Experiment	Error
C1	Vertical	Diameter	3.1	2.8	-0.3
		Height	5.0	4.5	-0.5
C2	Vertical	Diameter	2.6	2.9	0.3
		Height	4.1	4.6	0.5
C3	Vertical	Diameter	2.6	2.5	-0.1
		Height	6.1	5.8	-0.3

TABLE II
EXPERIMENTAL RESULTS FOR FIGURE (6)

Cylinder	Orientation	Type	Scene	Experiment	Error
C1	Vertical	Diameter	3.1	3.3	0.2
		Height	5.0	5.3	0.3
C2	Horizontal	Diameter	2.6	2.6	0.0
		Height	4.1	5.2	1.1
C3	Horizontal	Diameter	2.6	2.8	0.2
		Height	6.1	7.9	1.8

VI. Conclusion

We extended previous work on single view metrology and effectively utilized rotational symmetry for three dimensional reconstruction of repeated objects. The proposed method showed acceptable results with the scope of further improvements. The $360°$-rotational symmetry based virtual camera configuration framework can be further extended for view synthesis, motion analysis - segmentation and tracking. As well, proposed framework can be extended for vision tasks related to individual or repeated objects with reflection symmetry, uniform finite rotational symmetry and skewed symmetry.
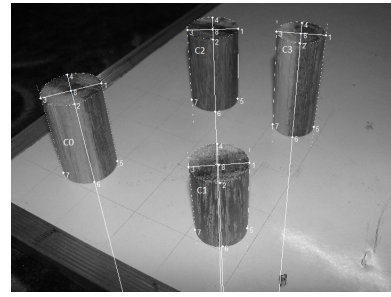

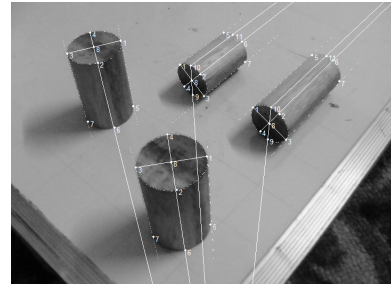Fig. 5.   Cylinders with Vertical Orientations

Fig. 6.   Cylinders with Random Orientation

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