CSL 866, Problem Sheet 2

1. Prove the statement of Radon's theorem.
2. If R_1 and R_2 are ε-samples of P_1 and P_2 where P_1 and P_2 are disjoint, then $R_1 \cup R_2$ is an ε sample of $P_1 \cup P_2$.
3. For a range space with discrepancy bounded by $\log^c n$ (polylog) rederive the bound for ε sample.
4. Prove the following theorem using discrepancy. Let (X, R) be a range space with shattering dimension d,where $|X| = n$, and let $0 < \varepsilon < 1$ and $0 < p < 1$ be given parameters. Then one can construct a set $N \subset X$ of size $O(\frac{d}{\varepsilon^2 p} \log \frac{d}{\varepsilon p}$ such that, for each range $r \in R$ of at least pn points, we have

$$|\frac{|r \cap N|}{|N|} - \frac{|r \cap X|}{|X|}| \leq \varepsilon \frac{|r \cap X|}{|X|}$$

Then N is called a relative (p, ε)-sample for (X, R).