CSL 356 Lecture 42 Nov 19

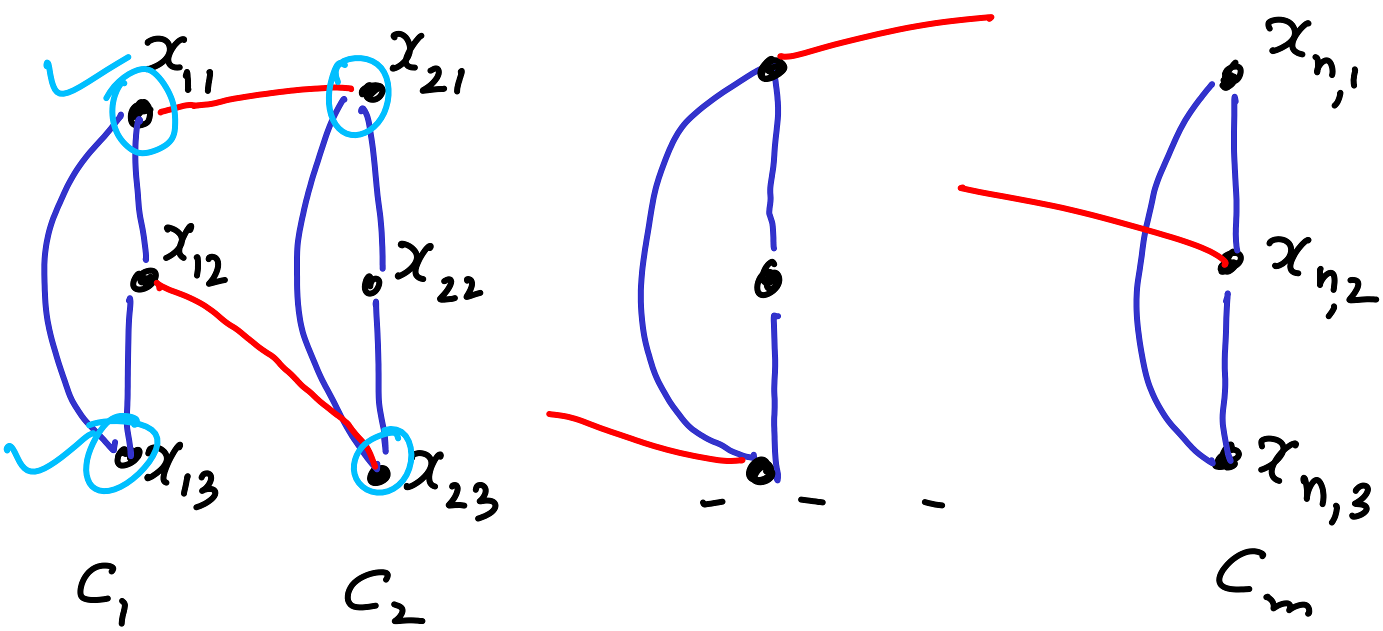
Reducing 3SAT to V.C.

$(x_1 \vee x_3 \vee \bar{x}_4) \wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \wedge \ldots$

T T F F T F

n variables $x_1, x_2, \cdots x_n$

m clauses $C_1 \quad C_2, \cdots \quad C_m$



$V = \{x_{11}, x_{12}, x_{1,3}, x_{2,1}, x_{2,2} \; \cdots \; x_{n,3}\}$

3m vertices

$E = \{(x_{11}, x_{12}), (x_{11}, x_{13}), (x_{1,2}, x_{1,3}) \cdots (x_{n,1}, x_{n,2})(x_{n,2}, x_{n,3}) \ldots$

$(x_{11}, x_{21}), (x_{12}, x_{2,3}) \cdots \}$

Claim The boolean formula F is satisfiable iff Graph $G = g(F)$ has a vertex cover of size $2m$ (m is # clauses)

1st part: If F is satisfiable then G has a VC of size $2m$

To construct a cover of size $2m$, leave out one of the literals set to True and include the other two in the cover

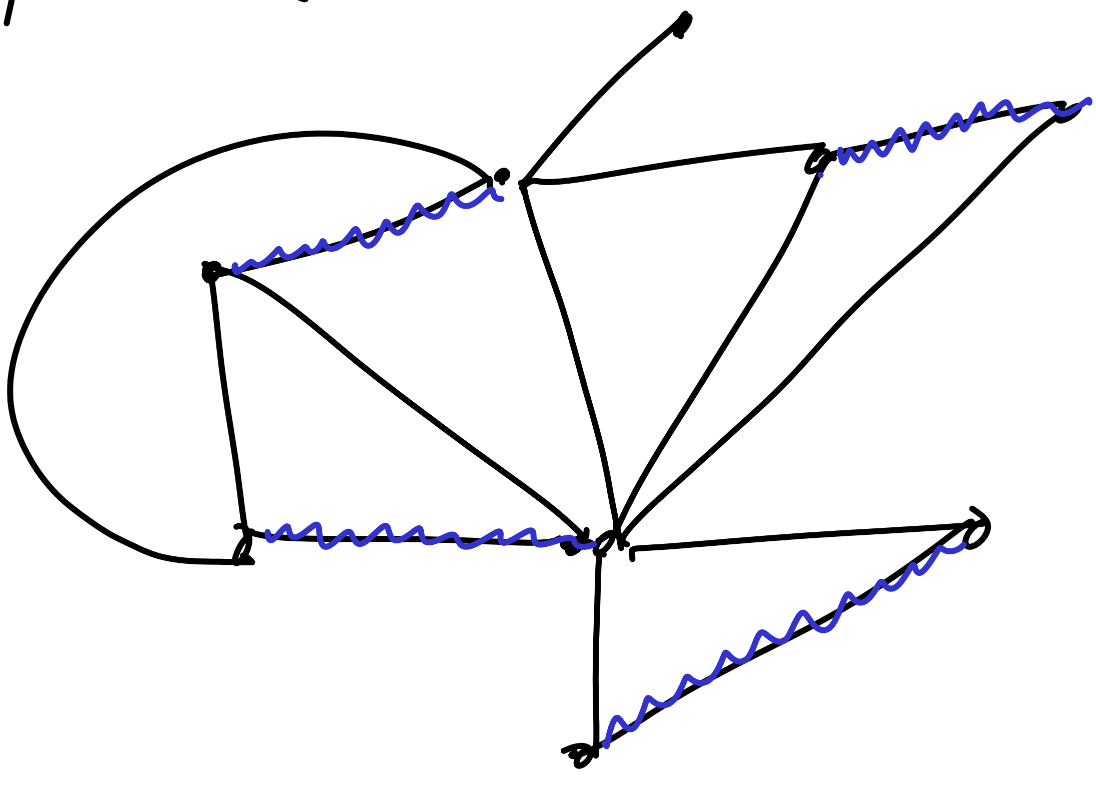
2nd part: If there is a VC of size $2m$, then F is satisfiable

Truth assignment: There must be exactly 2 vertices in the cover from each Δ. We assign the literal corresponding to the third vertex as True

What is the smallest VC for a given graph?

<u>Observation</u>

If we could solve the decision problem, i.e. is there a VC of size K, then we can also solve the optimisation problem.



G

<u>Maximal matching</u> : Suppose has K edges

Then V.C. $\geq K$

Choose both endpoints and call that subset W

<u>Claim</u>: W is a VC

(polytime)
This is an approximation algorithm with approximation 2.

$$\frac{\text{Size of our cover}}{\text{size of optimal cover}} \leq 2$$

→ Is there an approx algorithm for VC with approx < 2

→ It has been proved for many NP complete problems that approximation beyond a certain limit is not possible unless P = NP