COV 886, Problem Sheet 2

1. Given a range space $S = (X, R)$ with VC dimension d
   (i) What is the VC dimension of $(X, \bar{R})$ where $\bar{R} = \{r | X - r \in R\}$, i.e., the complement space.

2. Consider the range space $S = (X, \mathcal{R}$ where X is the set of points in the Eucidean d-dimensional space $\mathbb{E}^d$ and $\mathcal{R}$ is the set of closed half-spaces in $\mathbb{E}^d$.

   (i) What is the VC dimension of S ?
   Hint: You may want to make use of the following result called Radon's theorem. For a given set of $d + 2$ points in $\mathbb{E}^d$, there exists a disjoint partition of these points, say C, D such that $CH(C) \cap CH(D) \neq \phi$ where $CH()$ denotes the convex hull of a set of points.
   (ii) Is it easier to bound the *shattering dimension* of S ? What bound does it yield on the VC dimension ?

3. If R_1 and R_2 are ε-samples of P_1 and P_2 where P_1 and P_2 are disjoint, then $R_1 \cup R_2$ is an ε sample of $P_1 \cup P_2$.

4. For a range space with discrepancy bounded by $\log^c n$ (polylog) rederive the bound for ε sample.

5. Prove the following theorem using discrepancy. Let (X, R) be a range space with shattering dimension d,where $|X| = n$, and let $0 < \varepsilon < 1$ and $0 < p < 1$ be given parameters. Then one can construct a set $N \subset X$ of size $O(\frac{d}{\varepsilon^2 p} \log \frac{d}{\varepsilon p}$ such that, for each range $r \in R$ of at least pn points, we have

$$\left| \frac{|r \cap N|}{|N|} - \frac{|r \cap X|}{|X|} \right| \leq \varepsilon \frac{|r \cap X|}{|X|}$$

   Then N is called a relative (p, ε)-sample for (X, R).