**COL 757 Model Centric Algorithm Design**
**Problem Sheet 3**

1. For $M = O(B)$, what is the I-O complexity (number of block transfers) to transpose an $n \times n$ matrix ?
2. Design an efficient version of partition sort (quicksort with multiple pivots) for the external memory model with parameters $M$ and $B$. Show that it is comparable to mergesort.
   Hint: You may want to use the sampling lemma used for PRAM based partition sort.
3. Show that the average case lower bound for permutation is asymptotically similar to the worst-case bound.
4. A $k$-transposition permutes $n = k \cdot \ell$ elements as follows
   $x_1x_2x_3x_\ell x_{\ell+1}, x_{\ell+2}, \ldots x_{\ell \cdot k}, x_{\ell \cdot k+1} \ldots x_{\ell \cdot k+\ell}$ are mapped to $x_1, x_{\ell+1}, x_{2 \cdot \ell+1} \ldots x_{\ell k}, x_2, x_{\ell+2} \ldots x_{2\ell k} \ldots$
   Show how to do this in an external memory model using $O(\frac{n}{B} \log_{M/B} k)$ I-Os.
5. Describe a cache-efficient algorithm for computing the matrix product

   $$C^{n \times n} = A^{n \times n} \cdot A^{n \times n}$$

   for parameters $M, B$.
6. Describe an cache efficient impementation of shear sort in the external memory model with parameters $M, B$.
7. Describe a cache efficient algorithm for constructing planar convex hull of $n$ points in the external memory model.
8. Describe a cache efficient algorithm for finding the maximal elements of $n$ points on the plane in the external memory model.
9. Describe a cache efficient algorithm for computing *All nearest smaller value* problem in the I-O model.
10. Design a cache-oblivious algorithm for computing matrix transpose for the case $M \geq B^{3/2}$.