

**COL 757 Model Centric Algorithm Design
Problem Sheet 2**

1. Given two sorted sequences A and B of n elements, design an $O(\log \log n)$ time optimal speed up merging algorithm in the PRAM model.
2. Consider the following algorithm to sort an $n \times n$ array of numbers. Assume for simplicity that $n = 2^k$.
 - Sort the four $n/2 \times n/2$ subarrays recursively according to some indexing scheme.
 - Rotate every alternate rows of the smaller subarrays by $n/2$ positions right/left
 - Run 3 iterations of shearsort

Prove that the above algorithm correctly sorts and analyze the parallel running time in some appropriate model.

3. Describe an optimal speed up algorithm for list ranking using the idea of *random mate* discussed in class. Choose an appropriate PRAM model to get a $O(\log n)$ time $n/\log n$ processors algorithm.
4. Analyze the following variation of the parallel connectivity algorithm. Each directed tree is contracted to a star following the hooking step. Instead of the adjacency matrix, use a list or array data structure to implement the algorithm using $O(|E| + |V|)$ processors and polylog parallel time.
5. Consider a linear array of n processors p_i $1 \leq i \leq n$, that where initially processor p_i holds n_i packets. Moreover, $\sum_i n_i = n$, such that each processor is a destination of exactly one packet. Analyze the greedy routing algorithm with *furthest destination first* queue discipline for this problem giving rigorous and complete proofs.