COL 752 Geometric Algorithms

Minor 2 Sem I 2016 -17 Max 15, Time 1 hrs

Name ________________ **Entry No.** ________________ **Group**

Given a set S of n points in $\mathbb{R}^3$, design an efficient algorithm (preferably linear but no more than $O(n \log n)$) to find the smallest enclosing sphere of S.
Your algorithm can be randomized and you must provide detailed proof of correctness and running time.

Answer in the space provided below.

Use RIC where let $Minball(S, f)$ be defined as the smallest ball containing a set of S points of which f are known points $0 \leq f \leq 4$. The initial algorithm is called as $Minball(N, 0)$ since all the (maximum) 4 points on the boundary must be determined. Note that there can be 2,3 or 4 points on the surfce of the smallest ball containing all the points. If there are 2 or 3 points on the boundary then the ball can be determined by the points in dimensions 2 or 3 by considering the line (plane) passing through these points.

In the algorithm described below, we call $Minball(N, \phi)$.

Let $T(i, u)$ denote the expected running time of the algorithm $Minball$ with n points of which u are to be determined. Note that $0 \leq u \leq 4$.

Using backward analysis we can show that

$$T(i, u) \leq T(i-1, u-1) + \frac{u}{i} \cdot T(i-1, u-1) + O(1)$$

Using $T(i, 0) = O(1)$, we can show that $T(i, u) = O((u)!i)$ similar to the RIC based LP algorithm.

Procedure Compute the smallest enclosing ball(S, D)

Input S is a set of points in 3D where S is given as a random sequence ;
Let C_j denote the smallest ball containing the first j points defined by k points $k \leq 4$ $C_0 = D$;
while $j \leq n$ **do**
    if $p_{j+1} \notin C_j$ **then**
        if $D < 4$ **then**
            C_{j+1} is defined by $D \cup p_{j+1}$;
            $Minball(S - p_{j+1}, D \cup p_{j+1})$
        else
            Replace the last added point in D by p_{j+1}
        ;
    $j \leftarrow j + 1$;
Output (C_n) ;