Computational Geometry Lecture 38

Topic Geometric set cover (with bounded VC dimension)

Observation : When we double the weight of elements in some set A a β times, and we have a choice of doubling the weight of any of the elements, then the least total weight is obtained when the "doubling" is spread evenly across all elements.

Example : 2 elements e_1, e_2 and we double $a+b$ times

then verify $2^a + 2^b \geq 2 \cdot 2^{\frac{a+b}{2}}$

If we run i iterations of the sampling and doubling phase what is the minimum weight of the elements ($k = |OPT|$) in OPT ?

$$k \cdot 2^{\frac{i}{k}} \leq W_i = (1+\varepsilon)^i W_0$$

$$= (1+\varepsilon)^i \cdot m$$

$$\leq e^{\varepsilon i} \cdot m$$

Suppose $i = k \cdot q$

$$\Rightarrow \quad k \cdot 2^q \leq e^{\varepsilon k q} \cdot m$$

$$\log k + q \leq \log m + \varepsilon k q$$

$$q \underbrace{(1 - \varepsilon k)} \leq \log m - \log k$$

$$= \log\left(\frac{m}{k}\right)$$

Suppose $\varepsilon = \frac{1}{2k}$

$$\Rightarrow \quad q \leq 2 \cdot \log \frac{m}{k}$$

iterations where wt doubles is $O\left(k \cdot \log \frac{m}{k}\right)$

Size of cover (if we knew how to choose ε)

$$O\left(\delta^* \cdot k \log \underbrace{(\delta^* \cdot k)}_{\text{much smaller than } \log n}\right)$$

Shattering dim of S^*

$|OPT|$

$$O(OPT \cdot \log OPT)$$

Suppose we choose $\varepsilon = \frac{1}{4k}$ instead $\frac{1}{2k}$?

$$k \leq m \qquad k : 1, 2, ,$$

For the given $k = k_i$,
we try $O\left(k_i \log \frac{m}{k_i}\right)$ iteration
If it doesn't terminate, k_i is not the right value.

$$k = 1, 2, 4, \ldots$$