Computational Geometry Lecture 13

Topic Lower bounds - contd.

Important observation about linear decision trees.

Given any input point $p \in \mathbb{R}^n$ we follow some path from the root to some leaf node

$\Rightarrow$ Any decision tree also implies a partitioning of the $\mathbb{R}^n$ in the following way

"Every node t corresponds to some subset $W(t) \subset \mathbb{R}^n$, namely all points that pass thru t."

Every node is associated with a convex region. (intersection of linear inequalities)

The number of leaf nodes must exceed the number of connected-components of the solution space.

$\Rightarrow$ ht of linear decision tree is $\Omega(\log(\#W))$

where $\#W$ = no. of connected components in the solution space

$$\prod_{i \neq j} (x_i - x_j) = 0 \text{ iff answer is NO}$$

<u>Convex hulls</u> : relaxed version

(i) Enumerate (in any order) the corner points

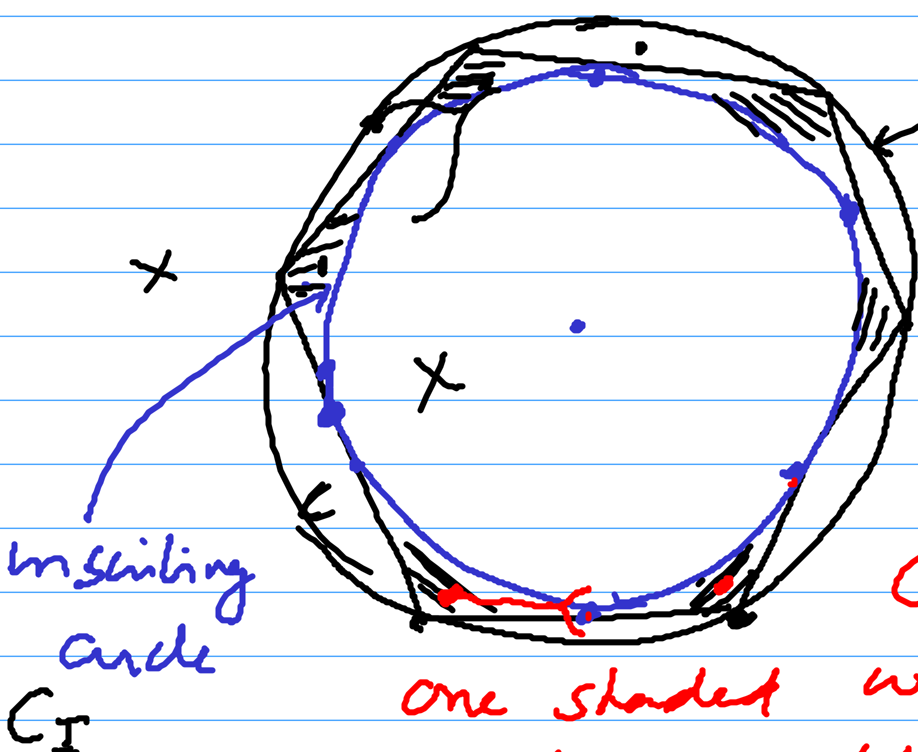
(ii) Given n points, do all points appear on the convex hull (Decision problem)

Version of convex hull that takes into account, the output size

P1

Given a set of n points on the plane and a number $h \leq n$, are there exactly h points on the convex hull?

Special case P2



Can we go from one shaded wedge to another using a path that completely lies in the shaded region?

$n-h$ points have to distributed to the h wedges h^{n-h} $\log(h^{n-h})$ $= (n-h)\log h$

P2 Fix the h points to be the regular h-gon and the remaining $n-h$ points are chosen arbitrarily in the annular region between C_I and C_c.

Are there h points on the convex hull?

Claim P2 $\propto$ P1

$\Rightarrow$ lower bound of P2 is a lower bound for P1