COL 702 Lecture 16 Sept 20

Job scheduling problem

	J_1	J_2	J_3	...	J_n
Time requirement	Δ_1	Δ_2	Δ_3		Δ_n
Deadline	d_1	d_2	d_3	..	d_n
Penalty	p_1	p_2	p_3	..	p_n

Δ_i, d_i are integral

Objective : Minimize penalty

Example $\quad \Delta_i = 1 \quad d_i = 1$

Then atmost one job can be completed within the deadline.

More general situation $\quad \Delta_i = 1$ but d_i s are distinct

Ex. 2 $\Delta_i = 1$

	J_1	J_2	J_3
d_i	1	2	2
p_i	5	2	8

Obs: 1. A job J_i with deadline d_i can be scheduled in any of the time slots $1, 2, \ldots d_i$ if we want to avoid penalty

2. Minimizing cumulative penalty of the jobs that miss their deadlines is the same as maximizing penalty of Jobs that are scheduled within deadlines

If we apply generic-greedy to this problem, then we should pick them in decreasing order of penalty and check feasibility using some efficient proced.

A <u>feasible</u> set of jobs $A \subset J$
can be scheduled within their respective deadlines

<u>Question</u> : Given a set of jobs A, is it feasible?

H.W assignment : complete with proof

The set of feasible jobs satisfies the subset system property

Is this subset system a matroid

① Exchange property ② Rank property

do these hold

Given subsets A, B of jobs s.t. A, B are feasible $|A|+1 = |B|$,
can we choose some $J_k \in B - A$ s.t. $A \cup \{J_k\}$ is feasible?

$A, B \in \mathcal{J}$: family of subsets that are feasible

	1	2	3	4	i		k	k+1
A	J_1	J_2	J_3		J_i		J_k	
B	J_1'	J_2'	J_3'					J'_{k+1}

Case 1 : $J'_{k+1} \notin A$ then $A \cup J'_{k+1}$ is feasible

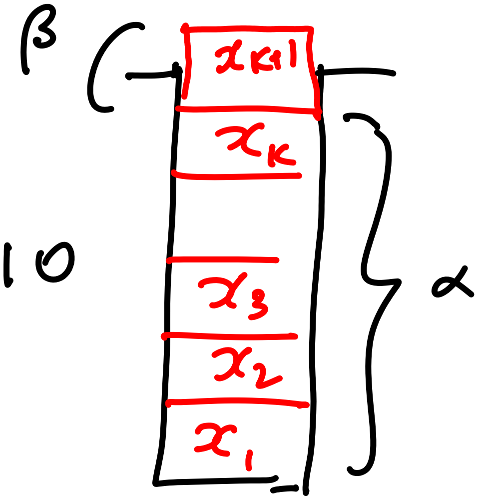
<u>Case 2</u> : $J'_{k+1} \in A$ i.e. $J'_{k+1} = J_i \in A$

By induction we can prove exchange property

What can we hope from greedy even when it is not a matroid?

Knapsack problem :

Choose the best profit/weight ratio



profit	20 16	10 22	15 4
weights	4	6	5 1
ratio	5 4	1.67 3	3 4

1st :

Picking strictly in decreasing order of ratios may not work

Suppose $x_1 \; x_2 \; x_3 \ldots x_k$ is the best soln when we pick by ratios but

$$\sum_{i=1}^{k+1} w(x_i) > B$$

$$\text{fits} \leftarrow \sum_{i=1}^{k} w(x_i) \leq B$$

Return as solution, $G \int \max \{ \{ w_1 \ldots w_k \}, w_{k+1} \}$

What can we claim about G
(vis a vis the optimal profit O)

$$\frac{G}{O} \geq 10\%, \; 20\% \; \ldots \; 50\%, \; 99\% ?$$

$\alpha + \beta \geq O \implies$ either α or β $\geq \frac{O}{2}$

$$\max\{\alpha, \beta\} \geq \frac{O}{2}$$

$\implies$ 50% optimum

— Matching can be solved in polynomial

Exercise : Run greedy - that produces a solution G

Compare with optimal O

Hint: