COL 702 Lecture 14 Sept 13

Optimization problem

1. Objective function
2. Constraints

Let $S = \{x_1, x_2 \ldots x_n\}$ be a set of objects having weights $w_1, w_2 \ldots w_n$

The set of all possible subsets is 2^S (power set)

Some of these subsets are "feasible" i.e. they satisfy the constraints

$\mathcal{J} \subset 2^S$ be the set of all feasible subsets

We want to choose $S^* \in \mathcal{J}$ which is the optimal soln:

For any subset $s \subset S$ $w(s) = \sum_{x \in s} w(x)$

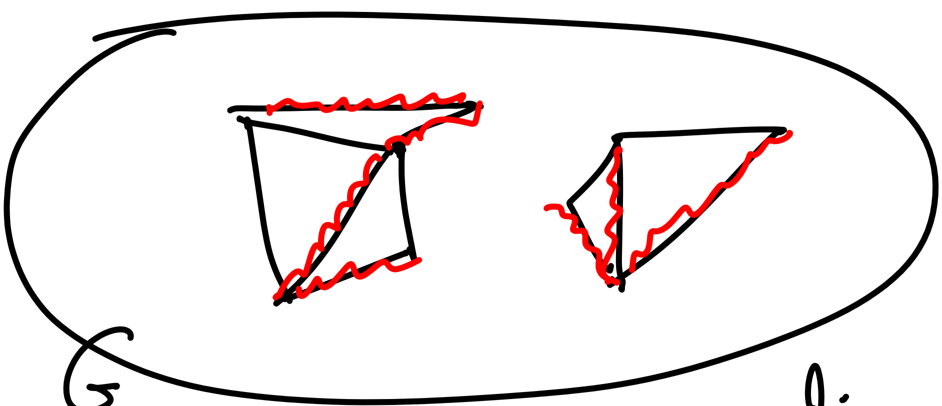
Further assume that for the maximization problem $w_i > 0$

Eg Knapsack, Set of objects and their profits (weights) and volumes V_i are given

Max sum of weights

$\mathcal{I}$: all subsets whose volume add up to a maximum of B

MSF : Maximum Spanning forest



given graph $G=(V,E)$
costs on edges

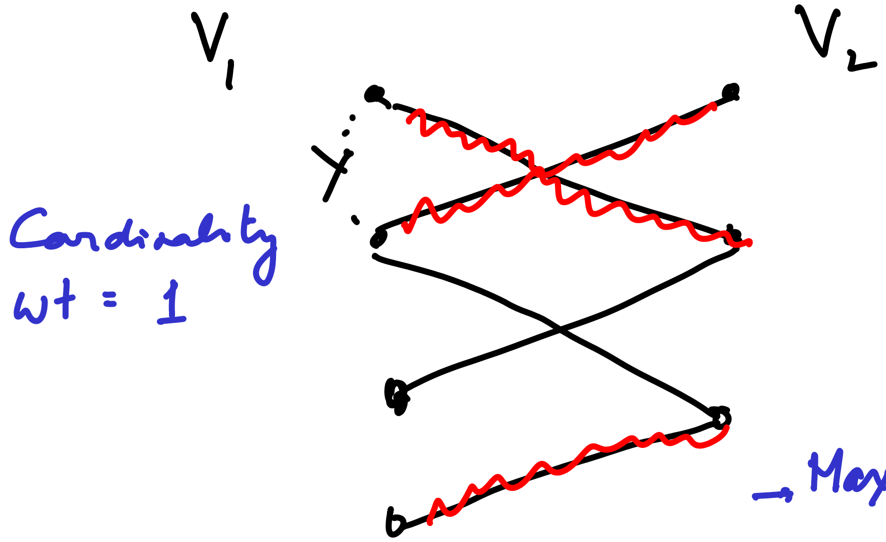
S: set of edges E

$\mathcal{I}$: $S \subset E$ s.t.

S doesn't contain a cycle

We want to choose the maximum wt forest

Matching problem / Assignment problem



Bipartite graph

$G = (V, E)$ is a weighted bipartite graph

→ Maximise the sum of wts of the matched edges

A feasible assignment is a subset of edges s.t. no two edge share a vertex

$\mathcal{I} : \{ S :$ if $e_1, e_2 \in S$, then they don't share an endpoint$\}$

Subset System : S_1, S_2 (are subsets of S)

s.t. $S_1 \subset S_2$

then if $S_2 \in \mathcal{I}$ then $S_1 \in \mathcal{I}$

Optimization Problem for Subset systems

$M = (S, \mathcal{I})$ and a wt function w

Simple-minded approach is greedy

Generic_greedy algorithm

Initialize $T = \phi$ (ϕ is always feasible)

Sort the objects in descending order of their wts, $x'_1, x'_2 \ldots x'_n$

For $i := 1$ to n do

Test for feasibility

if $T \cup \{x'_i\} \in \mathcal{J}$ then

Addition step

$T \leftarrow T \cup \{x'_i\}$

Output T

Running time $= \sum_{i=1}^{n} t_i$ $\quad t_i$: time to test and add the i^{th} object

Is T finally the optimum subset?

Doesn't work for matching, Knapsack

"may" work for MST

When does it work / doesn't work

The following are equivalent

Theorem : 0. The subset systems for which Generic-greedy works are called "matroids"

Properties that characterize matroids

1. Exchange Property : Consider two subsets S_1, S_2 $|S_2| > |S_1|$

Then there exists $x \in S_2 - S_1$ s.t. $S_1 \cup \{x\} \in \mathcal{I}$

NO REFERENCE TO WEIGHTS

2. Rank property : Let $A \subset S$

then if S_1, S_2 are maximal independent subsets in A - then $|S_1| = |S_2|$

We cannot add any further elements and still maintain feasibility.