COL 702, Tutorial Sheet 5

1. Give an $O(n+m)$ time i.e. linear time algorithm to determine if a 2-SAT formula is satisfiable.The formula has n variables and m clauses. (Hint: (**x V y**) is equivalent to $(\bar{x} \rightarrow y), (\bar{y} \rightarrow x)$.)

2. Prove that the following problems are NP-complete:

   (a) **CLIQUE**
   Instance: Graphs G=(V,E) and an integer $k \leq |V|$.
   Question: Is there a clique of size at least k in G ?
   (a) **SUBGRAPH ISOMORPHISM**
   Instance: Graphs G=(V,E) and H=(V',E').
   Question: Is H isomorphic to a subgraph of G?
   Definition: Graph $G_1 = (V_1, E_1)$ is **isomorphic** to $G_2 = (V_2, E_2)$ if there is a one-one onto function $f : V_1 \rightarrow V_2$ such that $(u, v) \in E_1$ iff $(f(u), f(v)) \in E_2$.
   (b) **DOMINATING SET**
   Instance: Graph G=(V,E) and positive integer k.
   Question: Is there $V' \subseteq V, |V'| = k$ such that each vertex u in (V-V') is adjacent to some vertex v in V'.

3. (a) Show that **PARTITION** is self-reducible, i.e. give a polynomial time algorithm to solve the search problem , given a subroutine for the decision problem.
   (b) Show that VERTEX COVER and CLIQUE are self reducible.

4. Show the existence of a co-NP complete problem. Provide all formal definitions.