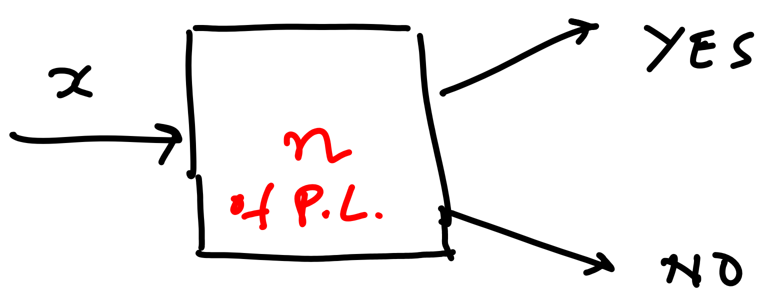
COL 352 Lecture 14 Jan 31

Given DFAs M_1 and M_2 how can we determine if $L(M_1) = L(M_2)$? (can be re. r_1 and r_2 also)

$$L_1 = L_2 \iff (L_1 - L_2) \cup (L_2 - L_1) = \phi$$

$L_1 - L_2 = L_1 \cap \overline{L_2}$ If L_1, L_2 reg then $L_1 - L_2$ is reg.

So can we test if $L(M) = \phi$ for a given M



Try all possible strings $x \in \Sigma^*$ $|x| < n$

If some string is accepted, clearly $L(M) \neq \phi$

If no string is accepted declare $L(M) = \phi$

<u>Proof</u> Suppose $x_1 \in L(M)$ and $|x_1| \geq n$ and among all such strings, this is the shortest.

From P.L. $x_1 = uvw$ and $uw \in L$
$\Rightarrow$ there is a smaller string, so contradiction.

Prob 5 Tut sheet 1

Let L be a regular language
Then the language

$$L_1 = \{a_1 a_3 a_5 \cdots a_{2n-1} \mid a_1 a_2 a_3 \cdots a_{2n} \in L\}$$
$$a_i \in \Sigma$$

is regular / not regular

Suppose $\Sigma = \{0,1\}$
and say L consists of the string

$L = \{ 110100011,\ 11010\ \ 100100,\ w_1 x w_2 x \cdots w_k x$
$w_1\ w_2\ w_3\ w_4\ w_k$

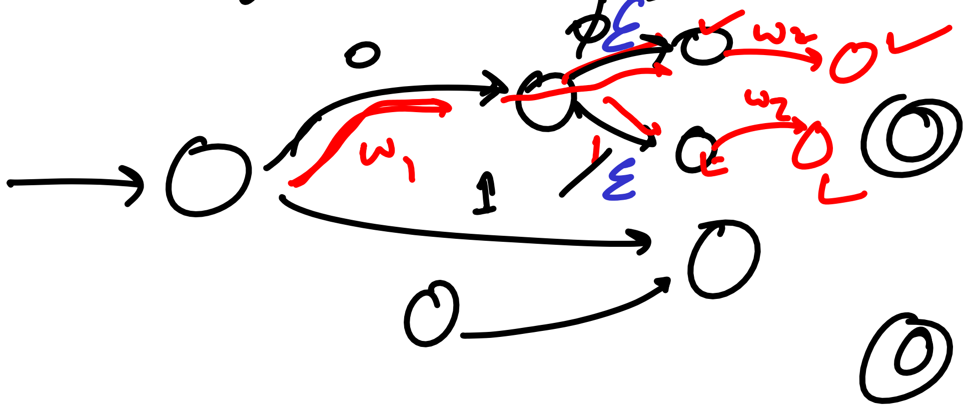
$L_1 = \{1001,\ 100\ ,$

Can we use PL to prove that some language L is regular?

If L is regular $\Rightarrow$ ( )

??

We have DFA to recognise L



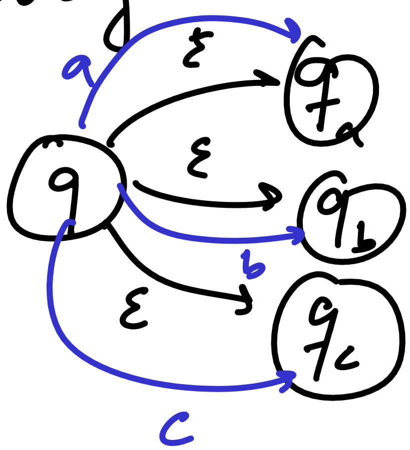
If $w_1 w_2 w_3 \cdots w_k \in L_1$ then there

$\exists \quad x_1, x_2, x_3 \dots x_k \qquad x_i \in \Sigma$

s.t. $\quad w_1 x_1 w_2 x_2 \cdots w_k x_k \in L$

$\iff \quad \hat{\delta}(q_0, w_1 x_1 w_2 x_2 \cdots w_k x_k) \in F$

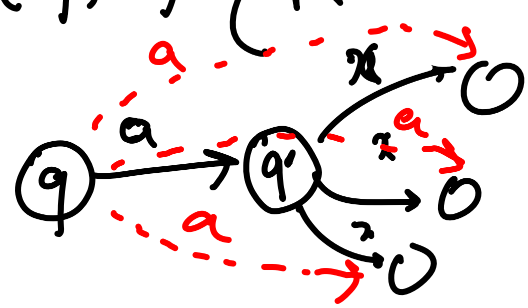
For every state q M, say q



every thing is accepted

new transition function

$\delta'(q, a) = \{ p \mid p = \delta(q, ax) \quad x \in \Sigma \}$



Claim 1 If $w_1 w_2 w_3 \cdots w_k \in L(M')$

then $\exists\ x_1 x_2 x_3 \ldots x_k \mid$

$w_1 x_1 w_2 x_2 \ldots w_k x_k \in L(M)$

Claim 2 If $w_1 x_1 w_2 x_2 \ldots w_k x_k \in L(M)$

$\Rightarrow w_1 w_2 w_3 \cdots w_k \in L(M')$