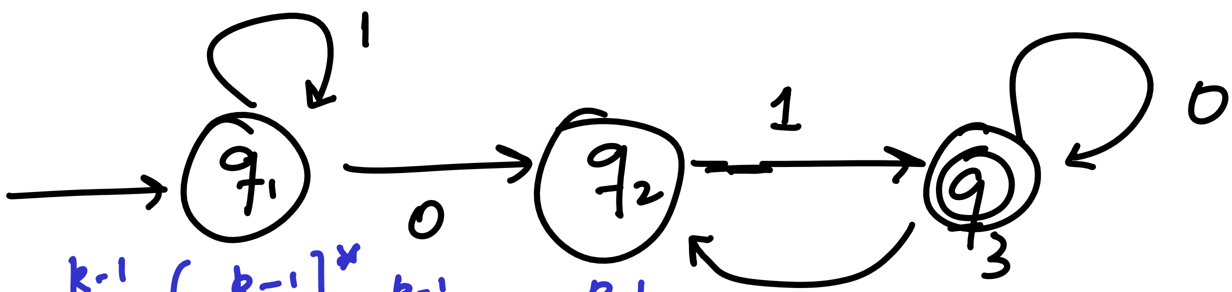
Lecture 12 COL 352 Jan 30



$$R_{ij}^{(k)} = R_{ik}^{k-1}\left[R_{kk}^{k-1}\right]^* R_{kj}^{k-1} + R_{ij}^{k-1}$$

$$1^*01\,(0+11)^*$$

	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
$K=0$	$1+\varepsilon$	0	ϕ	ϕ	ε	1	ϕ	1	$0+\varepsilon$
1 q_1 is allowed	$R_{11}^{(0)} \cdot (R_{11}^{0})^* \cdot R_{11}^{(0)} + R_{11}$ $(1+\varepsilon)\cdot 1^* \cdot (1+\varepsilon)$ $= 1^*$	$R_{11}^{0} \cdot (R_{11}^{0})^* \cdot R_{12}^{0} + R_{11}^{0}$ $(1+\varepsilon)\cdot(1+\varepsilon)^*\cdot 0 + 0$ $= 1^*0$	ϕ	ϕ	ε	1	ϕ	1	$0+\varepsilon$
2	1^*	1^*0	1^*01	ϕ	ε	1	ϕ	1	$0+\varepsilon+11$

$$(1+\varepsilon)^* = \varepsilon + (1+\varepsilon) + (1+\varepsilon)(1+\varepsilon) + \cdots (1+\varepsilon)^i$$

$$= \varepsilon + 1 + \varepsilon + 1 + \varepsilon + \varepsilon + \underbrace{\varepsilon^2}_{\varepsilon} + \cdots$$

$$= 1^*$$

Summary

1. Defined DFA / NFA

   Regular Language

   DFA $\sim$ NFA

2. R.e.

   R.e. $\sim$ DFA

What languages are not regular?

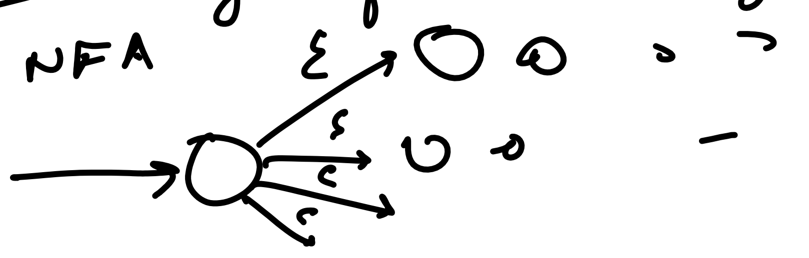
1. There exists languages that are not regular.

$$L = \{ 0^i 1^i \mid i \geq 0 \}$$

$$\{ \varepsilon + 01 + 0011 + 000111 + \cdots 0^{100} 1^{100} \}$$

__Claim__ Any finite language is regular

NFA



Claim : If L_1 is regular and L_2 is regular so is $L_1 \cup L_2$

r_1 r_2

$r_1 + r_2$ is regular

$L_1 \cup \boxed{L_2 \cup L_3 \quad L_k}$

Claim If L_1, L_2 are regular so is $L_1 \cdot L_2$

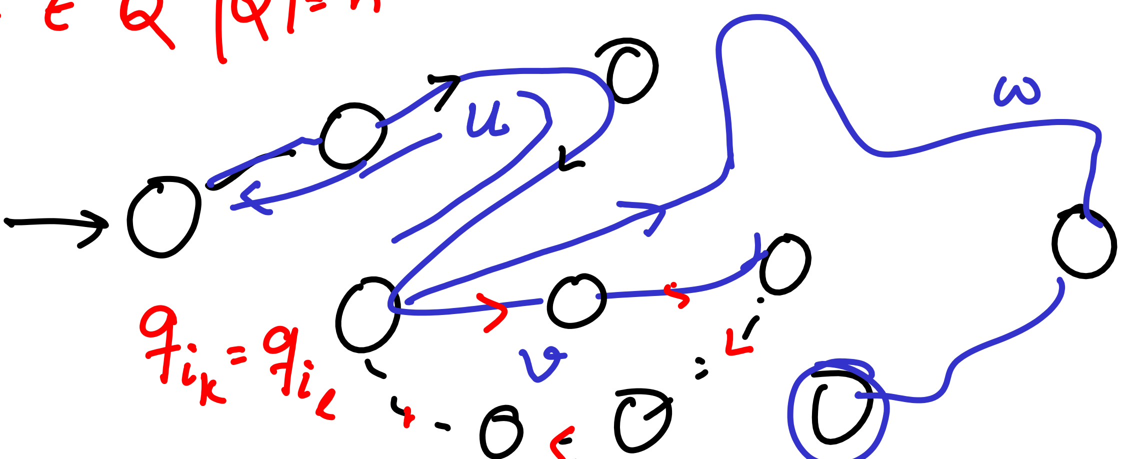
Claim If L is regular, so is L^*

Suppose we have a DFA M for a regular language L with n states (including initial state). Given a string of length n, $w_1 w_2 w_3 \ldots w_n$, $w_i \in \Sigma$, we have sequence of state transitions

$$q_0 = q_{i_1} \xrightarrow{w_1} q_{i_2} \xrightarrow{w_2} q_{i_3} \; \boxed{q_{i_k}} \cdots \overset{=}{} \boxed{q_{i_l}} \quad q_{i_{n+1}}$$

$\delta(q_{i_1}, w_1) = q_{i_2}$

$q_{i_j} \in Q \quad |Q| = n$



$x = w_1 w_2 \ldots w_n \in L$

s.t. $u v^i w \in L \quad i \geq 0$

Pumping Lemma

Given any string x of length $\geq n$ (# states) s.t. $x \in L$ we can partition $x = u v w$ s.t. $u v^i w \in L$ for $i \geq 0$. $|v| \geq 1 \quad |u \cdot v| \leq n$