COL 352 Lecture 4 Jan 11

Σ : finite alphabet

Σ^* : all finite length strings over Σ including ϵ (zero length)

Language $L \subseteq \Sigma^*$

We are interested in membership question for a given language L.

string $s = x_1 x_2 x_3 \cdots x_n$

- We can store at most some "constant" no. of input symbols
- We can scan the input only once

$\Sigma = \{0, 1\}$

$L_1 = \{ x \in \{0,1\}^* \mid \#\ 0\text{'s is divisible by } 3 \}$

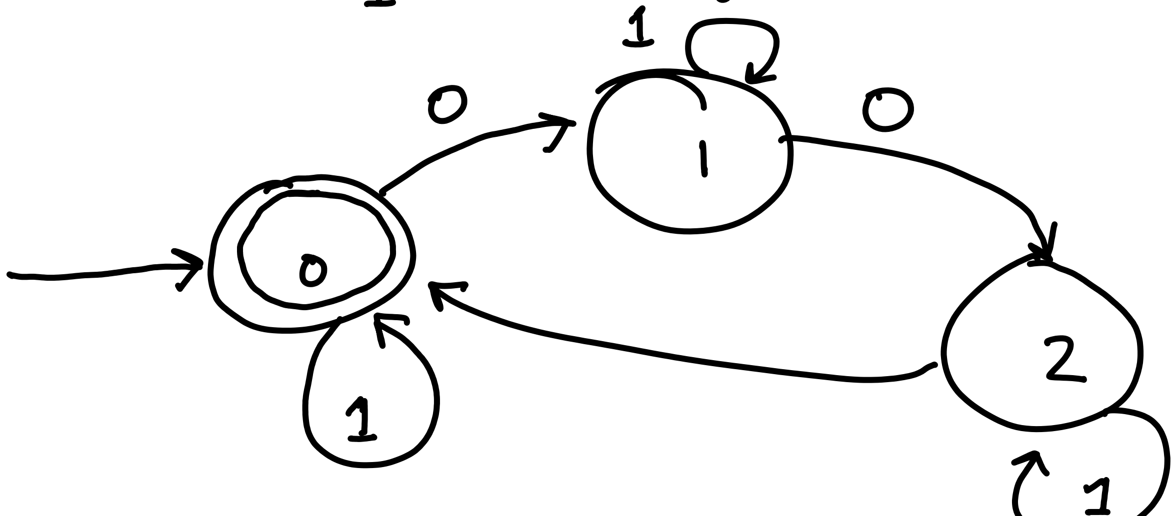
0 1 1 0 1 0 1 1 1 0 1 0 1

Idea: Keep track of the "mod" class of # 0's mod 3. 0, 1, 2

Initially we are in mod class [0]

next symbol $\xrightarrow{0}$ $[x] \to [(x+1) \bmod 3]$

next symbol $\xrightarrow{1}$ no change in mod class



1. There are finite 'states' (correspond to mod 3 classes)

   doesnot depend on length of string

2. What happen when the input symbol is 0, 1

Finite State Automaton / Machine

Σ : alphabet

Q : set of finite states q_1, q_2, q_3

q_0 : $q_0 \in Q$ initial state

F : $F \subseteq Q$ final states

$\delta : Q \times \Sigma \to Q$ transition function

$\{ (q_1, 0, q_2) \; (q_1, 1, q_3) \}$

$M = (\Sigma, Q, q_0, F, \delta)$

M corresponds to some language L

$L(M) = \{ x \in \Sigma^* \mid \delta^*(q_0, x) \in F \}$

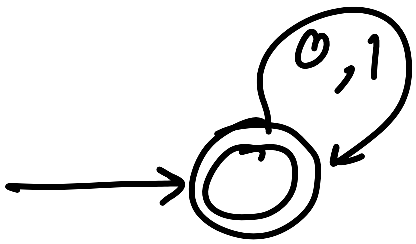
$\delta^*(q, a) = \delta(q, a) \qquad a \in \Sigma$

$\delta^*(q, ax) \quad x \in \Sigma^* \quad = \delta^*(\delta(q,a), x)$

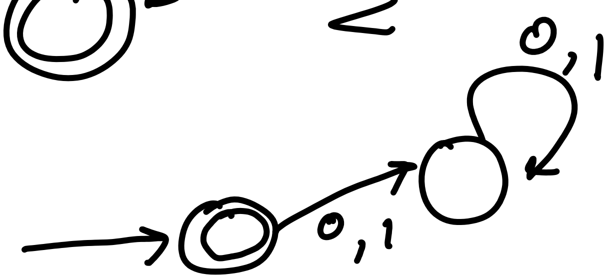
$\delta^*(q, \epsilon) : q$

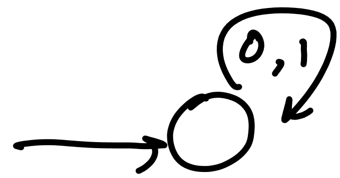
Given some language description L, design a FSA M s.t.

$L(M) = L$ (Nothing more nothing less)



Σ^*





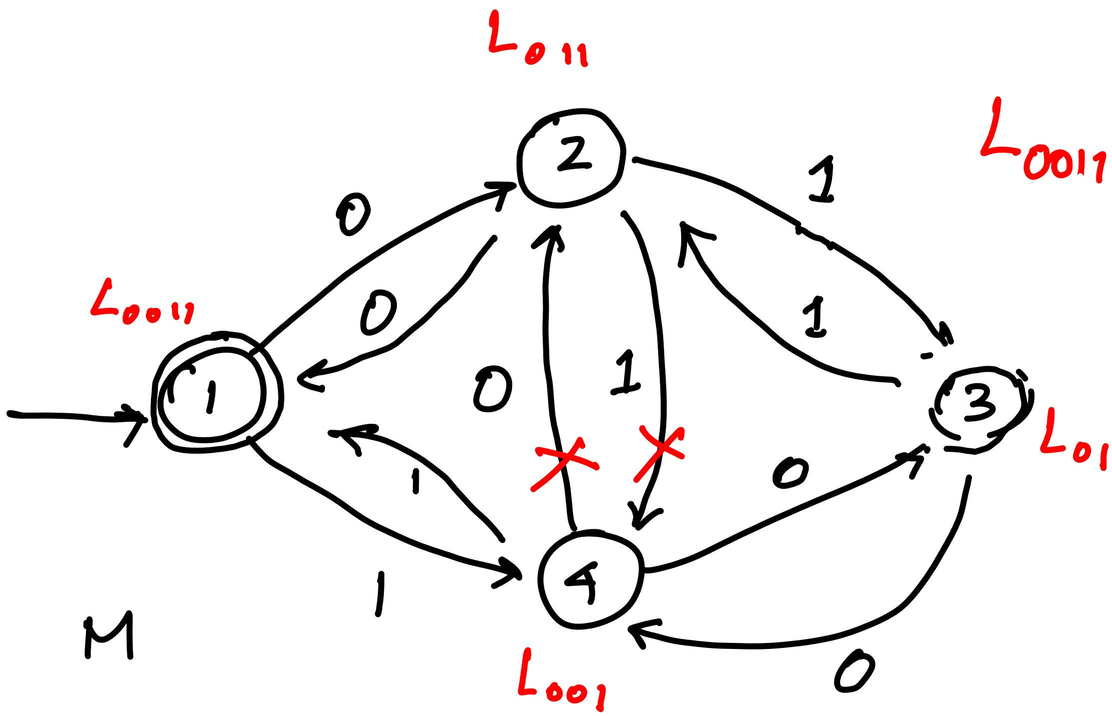
ϕ

empty language

$Q \backslash \Sigma$	a_1	a_2	a_3
q_0	q_2	q_1	
q_1			
q_2			

Given a machine M, show that

$L(M) = \{ \quad \}$



L_{0011} = {even 0's and even 1's}

What strings does this machine accept?

0 1 0 1 0 1

<u>Claim</u>: For all strings $w \in \{0,1\}^*$

$w \in L(M)$ iff w has even #0's and even #1's

(i) $w \in L(M) \Rightarrow w$ has even #0's even 1's

$\hat{\delta}(q_0, w) = q_1 \Rightarrow w \in L_{0011}$

(ii) If w has even #0's even 1's $\Rightarrow w \in L(M)$

$w \in L_{0011}$

We will prove (i) by using Induction on length of string.

For all $n \geq 0$, statement (i) is true

For all $\omega \in \{0,1\}^*$ statement (i) is true for $|\omega| \geq 0$

<u>Base case</u>

$|\omega| = 0$ $\quad \epsilon \quad$ $\epsilon \in L(M)$ and

✓ is accepted $\quad \epsilon \in L_{0011}$

Suppose for all $\omega \in \{0,1\}^*$ $|\omega| = n$.

statement (i) is true

We want to show that (i) is true for $|\omega| = n+1$

$\omega = \omega' 0$ or $\omega' 1$ $\quad |\omega'| = n$

$$\hat{\delta}(q, \omega a) = \delta(\hat{\delta}(q, \omega), a)$$

$\omega' 0$ — $\in L_{0011}$ $\quad \omega' \notin L(M)$

$\omega' 0$ — $\notin L_{0011}$ $\quad \omega' \notin L(M)$

(i) $\hat{\delta}(q_1, \omega) = \begin{cases} q_1 & \Rightarrow \quad \omega \in L_{0011} \\ q_2 & \omega \in L_{011} \\ q_3 & \omega \in L_{01} \\ q_4 & \Leftarrow \quad \omega \in L_{001} \end{cases}$

By induction on length of the string w, we want to show that

if $w \in L_{0011}$ then $\hat{\delta}(q, w) = q_1$

$w \in L_{011}$ then $\hat{\delta}(q_0, w) = q_2$

$\vdots$

<u>Base case</u> $w = \varepsilon \in L_{0011}$ and it is in q_1

If $w \in L_{011}, L_{001}, L_{01}$ then there is nothing to prove

Inductive Step : $w = w'0$ or $w'1$

for w' we use induction hypoth

Assignment due by 12 noon on Fri

$L' = \{ w \in \{0,1\}^* \mid$ the third last symbol is $1 \}$

$\downarrow$

0 0 0 0 1 0 ✗

0 1 0 1 ✓