

Post Correspondence Problem (PCP)  
is not recursive

$$L_u \leq_f L_{PCP}$$

	A	B
⊙	$s_1$	$w_1$
⊙	$s_2$	$w_2$
⋮		
⊙	$s_k$	$w_k$

$$f(\langle M, w \rangle) = (A, B)$$

s.t.  $A, B \in L_{PCP}$   
iff  $M$  accepts  $w$

$$s_{i_1} \cdot s_{i_2} \cdot s_{i_3} \cdot \dots \cdot s_{i_m} = w_{i_1} \cdot w_{i_2} \cdot w_{i_m}$$

$$i_j \in \{1, \dots, k\}$$

# Time and Space Complexity

Time complexity  $T(n)$   $n$  is the length of input string

$k$ -tape TM  $k \geq 1$

The TM always halt (recursive languages)

For a given TM  $M$ , the time complexity is  $T(n)$  if the maximum no of steps taken by  $M$  on any input of size  $n$  is  $T(n)$ .  
(for acceptance)

A language  $L$  is in the time complexity class  $T(n)$  if there is a TM that accepts  $L$  has time complexity  $T_k(n)$  ( $k$ -tapes)

$T(n)$  is at least  $n$  (by convention)

Unless the entire input is read, the language / problem is not considered interesting

A similar definition also holds for non-deterministic TM.

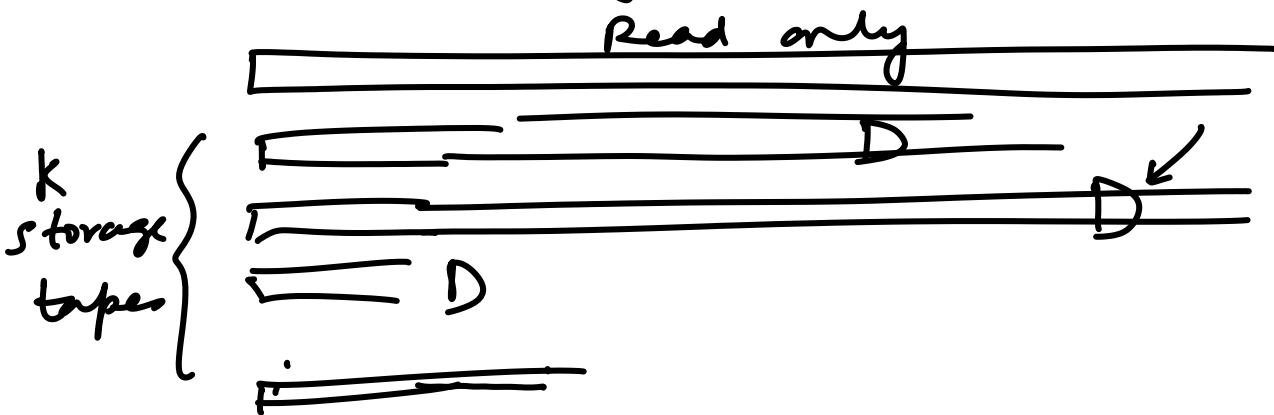


$$T(n) = \max_{\text{all inputs of length } n} \max_{\text{all choices for a fixed input}} T(n)$$

## Space Complexity

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k-tape TM but we have a read-only input tape



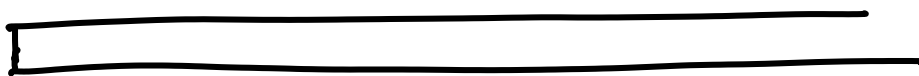
Max { Rightmost head position in any of the k storage tapes } not including input tape

Consider  $L_{Pal} = \left\{ w c w^R \mid w \in \Sigma^*, c \text{ is special char} \right\}$

Time complexity  $L_{Pal}$



$k=2$



By copying  $w$  in tape 2 and then checking  $w$  and  $w^R$  by moving the heads in opposite direction  $T(n) = n$  for  $L_{Pal}$

Space complexity of  $L_{Pal}$

Read only



Space complexity using the previous approach is  $\frac{n}{2}$  (length of  $w$ )

By using counters we can compare the corresponding characters of  $w$  &  $w^R$

A counter takes  $\log n$  space

So space complexity is  $\boxed{3 \log n}$