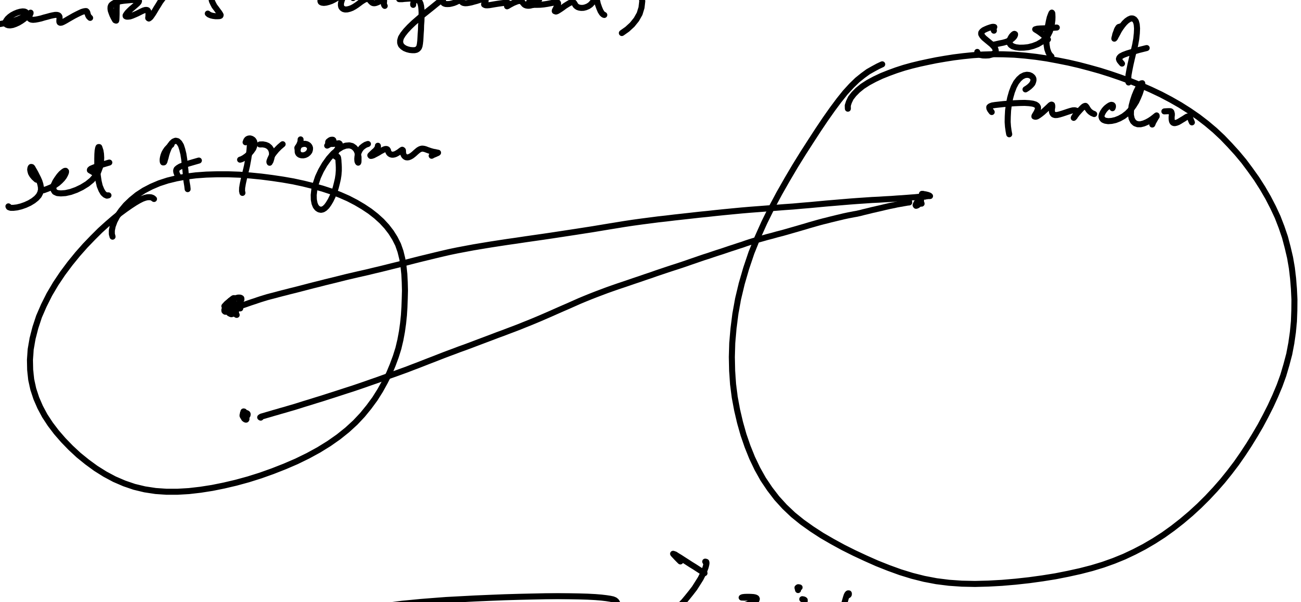
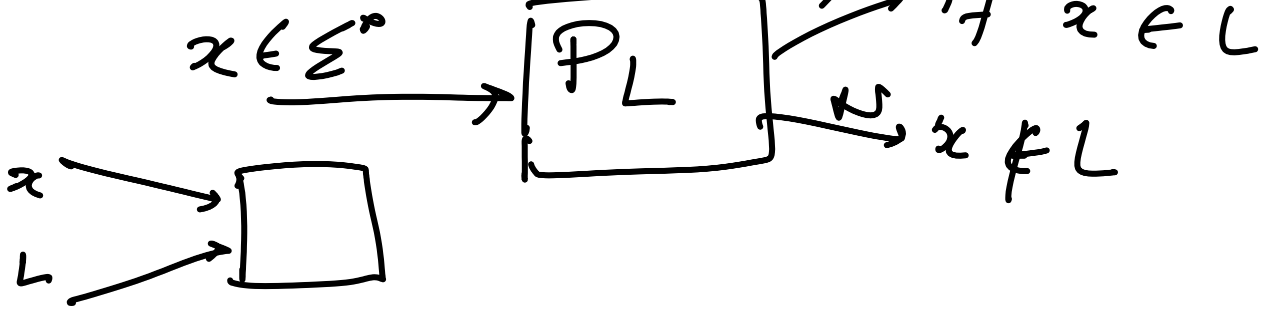
COL 352 · Lecture 3 Jan 7

$\#$ Possible programs (any language) is bounded by no. of strings ($\sim \#$ integers)

$\#$ Required functions (for membership problem) $\propto 2^{\mathbb{Z}}$

There is no bijection between a set and its powerset (Cantor's argument)





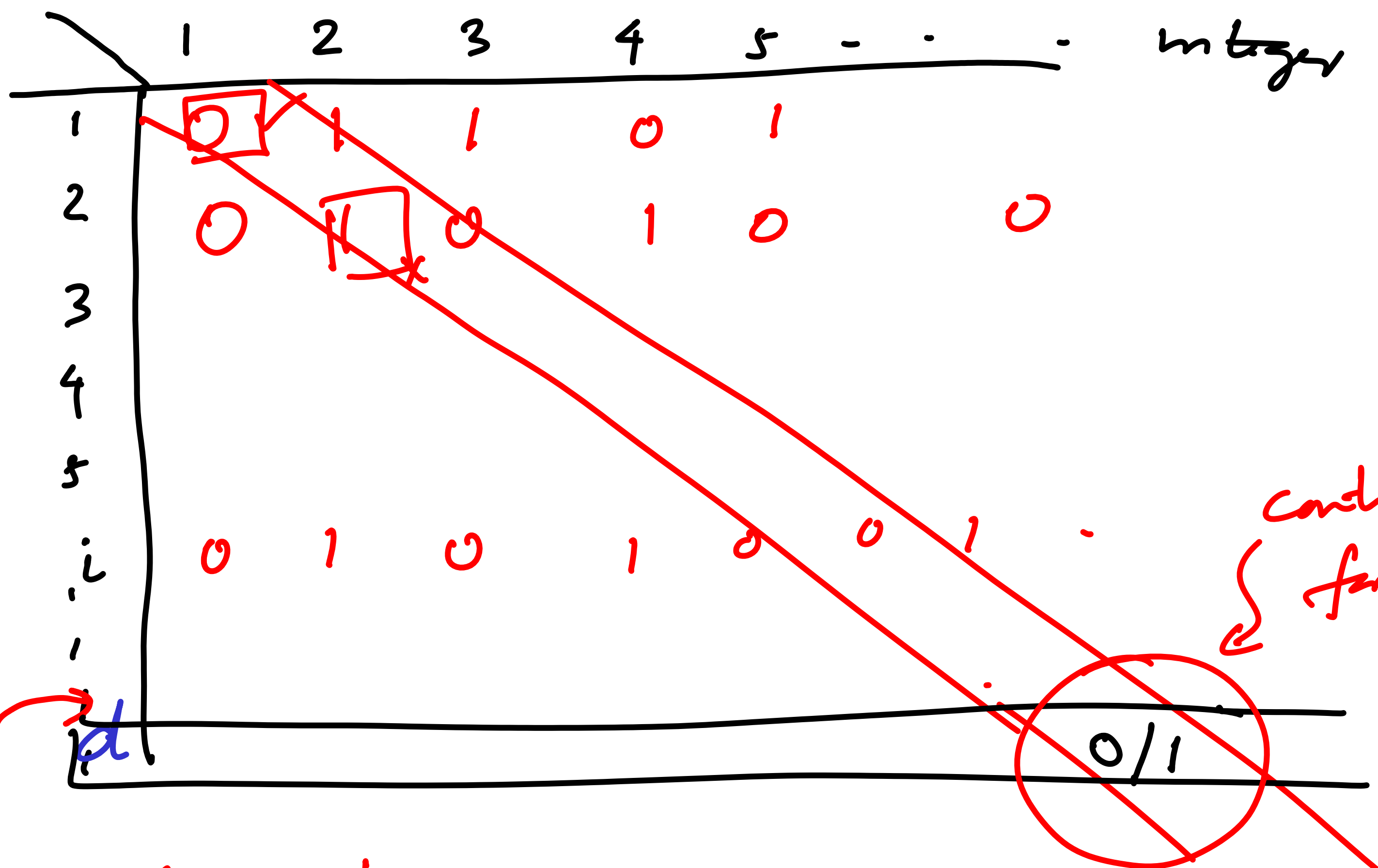
→ If set is finite, clearly true
$n < 2^n$

→ If set is infinite

Proof by contradiction

Suppose there is a bijection, i.e., the power set can be enumerated, we can express sets as

$S_1, S_2, S_3 \dots S_\alpha$ ← is an integer



contradiction for both

$S' : \{ i \mid i \notin S_i \}$

Suppose $S' = S_d$ Does $d \in S_d$?

The previous proof can be adapted for any set, not just $\mathbb{Z}$ and $2^{\mathbb{Z}}$

Let g be the bijection from $2^S \to S$

Then $S' = \{ x \mid x \notin g^{-1}(x) \}$

If $T = g^{-1}(x)$ then $g(T) = x$

Suppose $g(S') = \alpha$

Question Does $\alpha \in S'$?

$\alpha \in S'$ $\qquad\qquad$ $\alpha \notin S'$

$\Rightarrow \alpha \notin g^{-1}(\alpha) = S'$ $\qquad\qquad$ $\Rightarrow \alpha \in g^{-1}(\alpha) = S'$

Contradiction $\qquad\qquad$ Contradiction

Therefore g cannot exist that is 1-1 mapping from 2^S to S

Diagonalization

Assignment Set 0

Problems 2, 3, 4, 6

submit by next Thurs

Proof by Induction

Math Induction

To prove that a predicate P is true for all integers $0, 1, \ldots$ it suffices

$$\left[P(0) \wedge P(i) \Rightarrow P(i+1) \text{ for all } i \right]$$

$$\Rightarrow \forall i \geq 0 \; P(i)$$

Complete induction

$$\left[P(0) \wedge \left(P(0) \wedge P(1) \ldots P(i) \rightarrow P(i+1) \right] \right]$$

$$\Rightarrow \forall i \geq 0 \; P(i)$$