COL866: Quantum Computation and Information

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Administrative Information

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- Instructor
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Administrative Information

- Grading Scheme
  1. *Homework*: 10%
  2. *Quizzes* (announced) : 28%
  3. *Minor*: 25% each.
  4. *Major*: 35%
  5. *Attendance*: 2%
- Policy on cheating:
  - Anyone found using unfair means in the course will receive an **F** grade.

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- Textbook: Quantum Computation and Quantum Information by *Michael A. Nielsen and Isaac L. Chuang*.

- Gradescope: A paperless grading system.

- Course webpage: `http://www.cse.iitd.ac.in/~rjaiswal/Teaching/2024/COL866`.
  - The site will contain course information, references, problems. Please check this page regularly.

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- What is *quantum mechanics*?
    - Mathematical framework for constructing physical theories.

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- What should you expect to know by the end of the course?
  - Mathematical framework of for designing quantum algorithms and information processing.
  - Examples where quantum information processing systems have gone beyond classical ones.
    - Factoring, discrete logarithm, superdense coding, quantum search...

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  - Examples where quantum information processing systems have gone beyond classical ones.
    - Factoring, discrete logarithm, superdense coding, quantum search...
- This is not a Quantum Mechanics course!
  - We will start and build from a purely mathematical abstraction without going into the details of how the mathematical framework was arrived at or why such a framework might be reasonable.

- Church-Turing Thesis
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- What about quantum mechanical processes? Can they be simulated efficiently by Turing Machines?
  - There are examples where this is not known.
  - So, quantum computation may be the (only) candidate counterexample to the extended Church-Turing Thesis.

- Shannon's noiseless channel coding theorem
  - Quantifies the physical resources required to store the output of an information source.
- Shannon's noisy channel coding theorem
  - Quantifies the amount of information that is possible to reliably transmit through a noisy channel.
- What is the quantum analogue of the physical resource for encoding information? Qubit
- Some surprising results:
  - Superdense coding: Two classical bits can be communicated using a single quantum bit.
  - Distributed quantum computation: Quantum computers can require exponentially less communication to solve certain problems compared to classical computers.

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  - Quantum key distribution (Weisner,1960; Bennett and Brassard, 1984): Alice and Bob can communicate over a quantum channel to share a secret key even in presence of an adversary.
- Public key cryptography:
  - Alice and Bob both have a pair of public-private keys.
  - Messages are encoded using public key (that everyone knows) and can be decoded using the corresponding private key (that only the owner knows).
  - Such protocols exist. However, some popular ones become insecure if efficient algorithms for factoring and discrete logarithm problems are built.
  - Quantum algorithms: There are efficient quantum algorithms for both discrete logarithm and factoring.

- What is a qubit?
  - Qubit is to quantum computation as bit is to classical computation.
- Classical bit can be realised in real physical systems. Does it hold for qubits?
  - Yes but with a lot of *ifs* and *buts*. People would not have started talking about this concept if it were completely imaginary.
  - Since we do not have the expertise to go deeper into how qubits can be realised, we will treat it as a mathematical object.

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- Okay ... the classical bit has two states 0 and 1 (and that is pretty much the full description of the bit). Is qubit similar?

- What is a qubit? Quantum analogue of classical bit.
- Classical bit can be realised in real physical systems. Does it hold for qubits? We will work with yes.
- The classical bit has two states 0 and 1. Is qubit similar?
  - Yes and no. A qubit can be in states $|0\rangle$ and $|1\rangle$. However, these are not the only two states of the qubit.
  - A qubit can be in a superposition or linear combination of states:

    $$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

    where α and β are complex numbers.

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  - Yes and no. A qubit can be in states $|0\rangle$ and $|1\rangle$. However, these are not the only two states of the qubit.
  - A qubit can also be in a superposition or linear combination of states such as: $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$, where α and β are complex numbers.
- Then is it true that there are infinitely many possible states for a qubit?
  - Yes this is true.
- Can all these infinitely many states be recognised or measured? In other words, can one determine the state of a qubit (i.e., α, β)?
  - No. A measurement results in either 0 or 1 as output.
  - For a qubit in state $\alpha |0\rangle + \beta |1\rangle$, the probability of 0 is $|\alpha|^2$ and 1 is $|\beta|^2$ (*Note that this means* $|\alpha|^2 + |\beta|^2 = 1$)
  - Measurements changes the state of the qubit. If the measurement results in $x \in \{0, 1\}$, then the post-measurement state is $|x\rangle$.

End