- You may submit on Gradescope in groups of size at most 2.

There are 11 questions for a total of 40 points.

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1. (2 points) Can the following two-qubit state $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ be represented as $(\alpha|0\rangle+\beta|1\rangle)(\alpha'|0\rangle+\beta'|1\rangle)$? Give reasons.

Solution: (*You may type your answer in these "solution" boxes if you submit LaTeX*)

2. (2 points) Can there exist a single qubit gate with the following truth table? Give reasons.

Input	Output
$\|0\rangle$	$\frac{\sqrt{3}}{2}\|0\rangle+\frac{1}{2}\|1\rangle$
$\|1\rangle$	$\frac{1}{2}\|0\rangle+\frac{\sqrt{3}}{2}\|1\rangle$

3. (2 points) Show that there exist a single qubit gate with the following truth table? Give the matrix representation of such a gate.

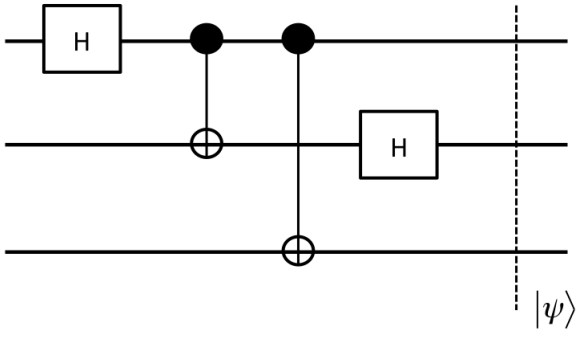
Input	Output
$\|0\rangle$	$\frac{\sqrt{3}}{2}\|0\rangle-\frac{1}{2}\|1\rangle$
$\|1\rangle$	$\frac{1}{2}\|0\rangle+\frac{\sqrt{3}}{2}\|1\rangle$

4. (4 points) Draw the classical circuit for computing the Boolean function $f:\{0,1\}^2\to\{0,1\}$ given by the following truth table.

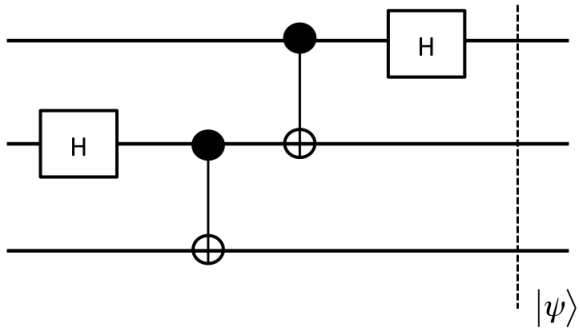
x	$f(x)$
00	1
01	0
10	1
11	0

Give the Quantum analogue of your classical circuit using Toffoli gates.

5. (4 points) Output $|\psi\rangle$ when the input to the circuit is $|000\rangle$. Output $|\psi\rangle$ when the input is $[\alpha|0\rangle+\beta|1\rangle]|00\rangle$.

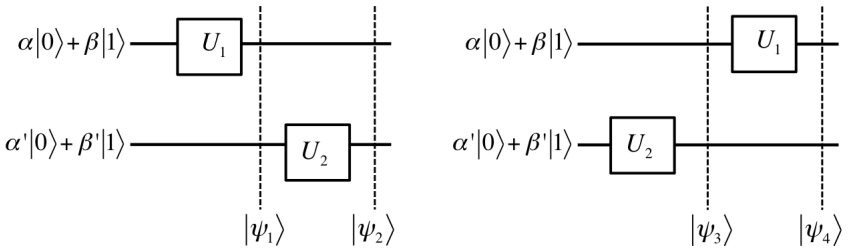


6. (4 points) Output $|\psi\rangle$ when the input to the circuit is $|000\rangle$. Output $|\psi\rangle$ when the input is $[\alpha|0\rangle + \beta|1\rangle]|00\rangle$.

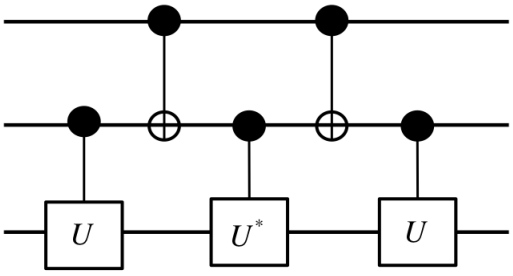


7. (2 points) Can you use a single qubit as a source of randomness? How?

8. (5 points) Let the matrix representation of gates U_1 and U_2 be $U_1 = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $U_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Give the states $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle$ in the circuits below.



9. (5 points) What is the input-output behaviour of the following circuit. (U^* *denotes conjugate transpose.*)



Input	Output
$\|00\rangle\|\psi\rangle$	
$\|01\rangle\|\psi\rangle$	
$\|10\rangle\|\psi\rangle$	
$\|11\rangle\|\psi\rangle$	

10. (5 points) Suppose you have two qubits in the bell state $\frac{|01\rangle-|10\rangle}{\sqrt{2}}$ and you apply the teleportation protocol to the first qubit. What is the result?

11. (5 points) Give the the intermediate states $|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle$ of the 3-qubit circuit given below. Show your calculations.