

There are 1 questions for a total of 12 points.

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- (12) 1. (*This problem is a subtle variant of the problem given in the theory part.*) A town has  $n$  residents labeled  $1, \dots, n$ . All  $n$  residents live along a single road. The town authorities suspect a virus outbreak and want to set up  $k$  testing centers along this road. They want to set up these  $k$  testing centers in locations that minimize the total distance that all the residents need to travel to get to their nearest testing center. You have been asked to design an algorithm for finding the optimal locations of the  $k$  testing centers.

Since all residents live along a single road, the location of a resident can be identified by the distance along the road from a single reference point (which can be thought of as the town's starting point). As input, you are given integer  $n$ , integer  $k$ , and the location of the residents in an integer array  $A[1..n]$  where  $A[i]$  denotes the location of resident  $i$ . Moreover,  $A[1] \leq A[2] \leq A[3] \leq \dots \leq A[n]$ . The goal is to find an integer array  $C[1..k]$  of locations such that the following quantity gets minimized:

$$\sum_{i=1}^n D(i), \text{ where } D(i) = \min_{j \in \{1, \dots, k\}} |A[i] - C[j]|$$

Here,  $|x - y|$  denotes the absolute value of the difference of numbers  $x$  and  $y$ . Note that  $D(i)$  denotes the distance resident  $i$  has to travel to get to the nearest testing center out of centers at  $C[1], \dots, C[k]$ .

(*For example, consider  $k = 2$  and  $A = [1, 2, 3, 7, 8, 9]$ . A solution for this case is  $C = [2, 8]$ . Note that for testing centers at locations 2 and 8, the total distance travelled by residents will be  $(1+0+1+1+0+1) = 4$ .)*

Design a DP algorithm for this problem that outputs the minimum achievable value of the total distance and the location of the test centers that gives the minimum total distance.

You have to implement your algorithm for the above problem. Your program should take input from a file named `input.txt` and should write the output in a file named `output.txt`. Your program should produce an output within 15 seconds for this assignment. The format for input and output files is as follows.

INPUT: The first line of the input gives the number of centers to be opened  $k$  (assume  $k \leq 10$ ). The second line of the input file gives the number of residents  $n$  (assume  $n \leq 1000$ ). The third line gives the location of the  $n$  residents in non-decreasing order, which are integers  $\leq 100000$ . Below is an example of an input file.

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```
2
6
1, 2, 3, 7, 8, 9
```

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OUTPUT: The first line of the output should contain the minimum achievable value of total distance. This should be followed by a line containing the locations of the  $k$  centers that give the minimum distance. This line should have locations in non-decreasing order. For example, consider the output file corresponding to the input file above:

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```
4
2, 8
```

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SUBMISSION INSTRUCTIONS: All your program files should be in a directory named `hw5_prog`. You will be asked to create a zip of this directory and submit this zip file. In this directory, there should be

a makefile that will compile your code (read about makefile on the net in case you do not know what it is). After running `make`, the directory should have an executable called `opcenters`. This, when executed, should read the input file (`input.txt`) and write the answer in the output file (`output.txt`).