COL702: Advanced Data Structures and Algorithms

(Semester-I-2022-23)

Grading

Grading component	**#**	**%**
Homework	5	20% (best 4 out of 5)
Quiz	5	20% (best 4 out of 5)
Minor	1	20%
Final	1	35%
Comprehension quiz	See below	5%

- *Comprehension quizzes are online quizzes conducted through **Moodle/Gradescope** that students are supposed to take for a better understanding of the material.*

Logistics

Grading component	**Comments**
Homework	• Can be done in groups of size at most 2. • Submissions to be made over Gradescope. Please avoid missing deadlines. • We will prefer if you submit typed solutions. Latex source files will be provided. • May include a programming component.
Quiz	• Conducted using **online** Gradescope feature or in class. • We may create multiple versions of the quiz.
Comprehension quiz	• One quiz per lecture component. There may be around 20 such quiz. • We prefer if you take the quiz soon after the lecture component has been completed. However, we will have a flexible deadline.

Important points

- Please register on Piazza (no code required).
- You will be registered on Gradescope by syncing with Moodle.
- Course webpage:
  - *https://www.cse.iitd.ac.in/~rjaiswal/Teaching/2022/COL702/*
- Textbook
  - -Algorithms by *Dasgupta, Papadimitriou, and Vazirani.*
  - Algorithm Design by *Jon Kleinberg and Eva Tardos.*
- Audit:
  - You will need to score at least **40%** for Audit-pass. You should plan to take all the grading components even if you audit.

Analyzing algorithms

A royal mathematical challenge (1202):

Suppose that rabbits take exactly one month to become fertile, after which they produce one child per month, forever. Starting with one rabbit, how many are there after n months?

Leonardo da Pisa, aka Fibonacci

The proliferation of rabbits

Rabbits take one month to become fertile, after which they produce one child per month, forever.

	Fertile	Not fertile
Initially		
One month		
Two months		
Three months		
Four months		
Five months		

The Fibonacci sequence

$F_1 = 1,$ $F_2 = 1,$ $F_n = F_{n-1} + F_{n-2}$

These numbers grow *very* fast: $F_{30} > 10^6$!

In fact, $F_n \approx 2^{0.694n} \approx 1.6^n$, exponential growth.

The Fibonacci sequence

$F_1 = 1$, $F_2 = 1$, $F_n = F_{n-1} + F_{n-2}$

Can you see why $F_n < 2^n$?

Computing Fibonacci numbers

```
function Fib1(n)
if n = 1 return 1
if n = 2 return 1
return Fib1(n-1) + Fib1(n-2)
```

A *recursive* algorithm

Two questions we always ask about algorithms:

Does it work correctly?

How long does it take?

Running time analysis

```
function Fib1(n)
if n = 1 return 1
if n = 2 return 1
return Fib1(n-1) + Fib1(n-2)
```

Exponential time... how bad is this?

Eg. Computing F_{200} needs about 2^{140} operations.
How long does this take on a fast computer?

IBM Summit

Can perform up to 200 quadrillion (= 200 x 10^{15}) operations per second.

Is exponential time all that bad?

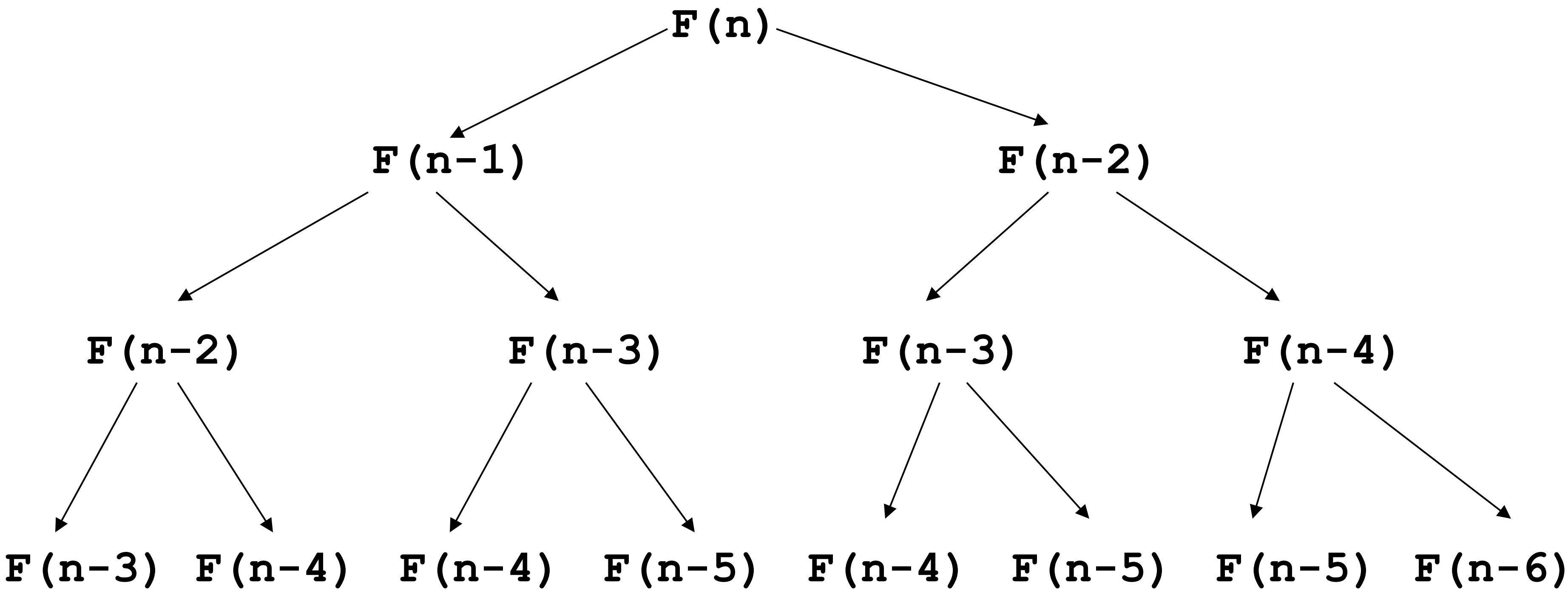
The Summit needs 2^{82} seconds for F_{200}.

Time in seconds	Interpretation
2^{10}	17 minutes
2^{20}	12 days
2^{30}	32 years
2^{40}	cave paintings
2^{45}	*homo erectus* discovers fire
2^{51}	extinction of dinosaurs
2^{57}	creation of Earth
2^{60}	origin of universe

Post mortem

What takes so long?
Let's unravel the recursion…

```
function Fib1(n)
if n = 1 return 1
if n = 2 return 1
return Fib1(n-1) + Fib1(n-2)
```



The same subproblems get solved over and over again!

A better algorithm

Subproblems: F_1, F_2, …, F_n. Solve them in order and *save their values*!

```
function Fib2(n)
Create an array fib[1..n]
fib[1] = 1
fib[2] = 1
for i = 3 to n:
    fib[i] = fib[i-1] + fib[i-2]
return fib[n]
```

[1] Does it return the correct answer?

[2] How fast is it?

Polynomial vs. exponential

Polynomial running times:

Exponential running times:

To an excellent first approximation:

polynomial is reasonable

exponential is not reasonable

This is one of the most fundamental dichotomies in the analysis of algorithms.