- The instructions are the same as in Homework 1–5.

There are 3 questions for a total of 75 points.

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1. Solve the following two problems:
   (a) (10 points) An edge in a network flow graph is called *downwards critical* if decreasing the capacity of this edge decreases the maximum flow in the network. Design an algorithm to find a downwards critical edge in a network. Give pseudocode, discuss running time and give proof of correctness.
   (b) (15 points) An edge in a network flow graph is called *upwards critical* if increasing the capacity of this edge increases the maximum flow in the network. Design an algorithm to find an upwards critical edge in a network in case there exists one. Give pseudocode, discuss running time and give proof of correctness.

2. (25 points) Design an algorithm that outputs a minimum vertex cover of a given bipartite graph $G = (L, R, E)$. Give pseudocode, discuss running time and give proof of correctness.

3. (25 points) You are given n pairs of integers $(d_1, d'_1), ..., (d_n, d'_n)$ such that $\forall i, d_i, d'_i \geq 0$. You have to design an algorithm that determines if there exists a directed graph $G = (\{1, ..., n\}, E)$ such that the in-degree of vertex i is d_i and the out-degree of vertex i is d'_i. Your algorithm should also output a graph with the given degree sequence, in case there exists one. Give pseudocode, discuss running time and give proof of correctness.